Problem 1: Construct truth tables for the statements:
(i) not \((P \lor Q)\)
(ii) \((\neg P) \land (\neg Q)\).
(iii) Are the statements ‘not \((P \lor Q)\)’ and ‘\((\neg P) \land (\neg Q)\)’ logically equivalent? Explain.

Problem 2: Do 2.5(i) on p. 20.

Problem 3: Prove that
(i) \(n^2 - 2n + 1 = 0 \iff n = 1\).
(ii) \(n^2 - 3n + 2 = 0 \iff (n = 1 \text{ or } n = 2)\).

Problem 4: Prove that ‘\((P \lor Q) \Rightarrow R\)’ and ‘\((P \Rightarrow R) \land (Q \Rightarrow R)\)’ are logically equivalent.

Problem 5: Define an integer \(a\) to be odd if there exists an integer \(b\) such that \(a = 2b+1\). Prove directly that the square of an odd integer is odd.

Problem 6: Using Axioms 3.1.2, do #3.8 on p. 29.

Problem 7: Using #5, prove by contradiction that for any integer \(n\),
\[
n^2 \text{ is even } \Rightarrow n \text{ is even}.
\]

Problem 8: Are the statements ‘\(P \Rightarrow (Q \lor R)\)’ and ‘\((P \land (\neg Q)) \Rightarrow R\)’ equivalent?

Problem 9: Prove the following. Let \(a\) and \(b\) be real numbers.
If \(ab = 0\), then \(a = 0\) or \(b = 0\).

Problem 10: Prove by induction on \(n\) that, for all positive integers \(n\),
\[
1^2 + 3^2 + \cdots + (2n - 1)^2 = \frac{n(2n-1)(2n+1)}{3}.
\]

Problem 11: Let \(u_1, u_2, u_3, \ldots\) denote the Fibonacci numbers, defined by \(u_1 = 1, u_2 = 1\) and \(u_{k+1} = u_{k-1} + u_k\) for \(k \geq 2\). Prove by induction on \(n\) that, for all positive integers \(n\),
\[
u_{4n} \text{ is divisible by 3}.
\]

Problem 12: Let \(x\) be a real number greater than \(-1\). Prove by induction that for each positive integer \(n\),
\[
(1 + x)^n \geq 1 + nx.
\]