Math 109. 2015 Summer Session I. Final. July 31. (10 points each question; 100 points total)

Instructions:

(1) Write your name and section number (AO1, A02, or A03) on your blue book

(2) Write clearly and give a reasonable amount of explanation.

Notations:

$\mathbb{Z}$ is the set of integers

$\mathbb{N} = \mathbb{Z}^+ = \{1, 2, 3, \ldots\}$ is the set of positive integers

$\mathbb{R}$ is the set of real numbers

$\mathbb{N}_n = \{1, 2, \ldots, n\}$

$\mathbb{Z}_n = \{0, 1, 2, \ldots, n - 1\}$ or $\mathbb{Z}_n = \{[0], [1], [2], \ldots, [n - 1]\}$

$a|b$ denotes $a$ divides $b$

$\text{Fun}(X,Y)$ denotes the set of functions from $X$ to $Y$, that is, $f \in \text{Fun}(X,Y)$ if and only if $f : X \to Y$ is a function.

Definitions:

An integer $p \geq 2$ is prime if its only positive divisors are 1 and $p$.

We say that $d$ is a perfect square if there exists $e \in \mathbb{Z}$ such that $e^2 = d$.

Integers $a$ and $b$ are congruent modulo $m$ (written as $a \equiv b \mod m$) if and only if $a - b$ is divisible by $m$, i.e., $m|(a - b)$.

Facts that you may use:

not $(P \lor Q) \iff$ (not $P$) and (not $Q$).

not $(P \land Q) \iff$ (not $P$) or (not $Q$).

If $d$ divides $a$ and $d$ divides $b$, then $d$ divides $ma + nb$ for any $m, n \in \mathbb{Z}$.

Let $g = \gcd(a, b)$. Then there exist $m, n \in \mathbb{Z}$ such that $g = ma + nb$ (this is a consequence of the Euclidean algorithm).

Division Theorem: If $a \in \mathbb{Z}$ and $b \in \mathbb{N}$, then there are unique $q, r \in \mathbb{Z}$ such that $a = bq + r$ and $0 \leq r < b$. 
