#23. One of the main things is to convert the word problem to solving math equations. Start with “(Assume constant acceleration.)”. So the 727 jet has

\[ a(t) = a = \text{constant}. \]

We don’t know \( a \), so this is something we need to find out.

Model: The 727 jet goes along a straight line on the runway. Since the derivative \( v'(t) \) of the velocity \( v(t) \) is the acceleration \( a(t) \) and the initial velocity \( v(0) \) is zero, we have

\[ v'(t) = a, \]
\[ v(0) = 0. \]

Finding this easy antiderivative, we get

\[ v(t) = at + v(0) = at. \]

Since we want to find how far the 727 jet has gone at time \( t \), we let \( s(t) \) be the position of the 727 jet. It is simplest to let \( s(0) = 0 \), so that \( s(t) \) is the distance travelled. Solving

\[ s'(t) = v(t) = at, \]
\[ s(0) = 0, \]

we get

\[ s(t) = \frac{a}{2} t^2 + s(0) = \frac{a}{2} t^2. \]

We want to find how far the 727 jet has travelled when it going 200 mph. We know that the time is 30 seconds. Since we want consistent units, we have all distances in miles and all times in hours. 30 seconds equals \( \frac{1}{120} \) hours. So

\[ 200 = v\left(\frac{1}{120}\right) = a \cdot \frac{1}{120}. \]

That is, the constant acceleration is

\[ a = 120 \cdot 200 = 24000 \text{ miles per hour}^2. \]

We can plug this into the formula for the distance travelled to get

\[ s(t) = \frac{24000}{2} t^2 = 12000t^2. \]

So the distance travelled at time \( t = \frac{1}{120} \) (this is when the 727 jet is going 200 mph) is

\[ s\left(\frac{1}{120}\right) = 12000 \cdot \left(\frac{1}{120}\right)^2 = \frac{100}{120} = \frac{5}{6} \text{ miles}. \]

So the runway has to be at least \( \frac{5}{6} \) of a mile long. □
The units we choose are: distances will be in feet and time will be in seconds. So 70 mph is \( 70 \cdot \frac{5280}{3600} = \frac{308}{3} = 102.6 \) ft/sec (because there are 5280 feet in 1 mile and 3600 seconds in 1 hour). This is the initial velocity, so we write:

\[ v(0) = \frac{308}{3}. \]

Since the Acura NSX has constant acceleration while it is braking, we have

\[ v(t) = \frac{308}{3} + at. \]

Again, for simplicity, we choose \( s(t) \) so that \( s(0) = 0 \). Then

\[ s(t) = \frac{308}{3}t + \frac{a}{2}t^2. \]

The Acura NSX stops at distance 157 feet. We first want to find out what time this is. We have two equations for the two unknowns \( a \) and \( t \) (the velocity is zero and the distance is 157):

\[ v(t) = 0, \quad s(t) = 157. \]

That is,

\[ 0 = v(t) = \frac{308}{3} + at, \]

which tells us \( at = -\frac{308}{3} \), and

\[ 157 = s(t) = \frac{308}{3}t + \frac{a}{2}t^2 = \frac{308}{3}t + \frac{at}{2}. \]

So

\[ 157 = \frac{308}{3}t + \frac{308}{3} \cdot \frac{1}{2}t = \frac{308}{2} \cdot \frac{3}{3}t, \]

which implies the time at which the Acura NSX stops is:

\[ t = \frac{157 \cdot 6}{308}. \]

From \( at = -\frac{308}{3} \), we see that the acceleration is:

\[ a = -\frac{308}{3t} = -\frac{308}{3 \cdot \frac{157 \cdot 6}{308}} = -33.568 \text{ ft/sec}^2. \]