Math 10B  Steven’s F12 Final Hints

1. 
   (a) \( \int_a^b (2f(x) + 3g(x)) \, dx = 2 \int_a^b f(x) \, dx + 3 \int_a^b g(x) \, dx \).
   
   (b) If \( f \) is an odd function, then \( \int_{-a}^0 f(x) \, dx = - \int_0^a f(x) \, dx \).
   
   (c) \( G(b) = G(0) + \int_0^b G'(x) \, dx \) (FTC).

2. \( \frac{ax^2 + bx + c}{x} = ax + b + \frac{c}{x} \) (but no way to simplify \( \frac{x}{ax^2 + bx + c} \)).

3. FTC and Chain Rule: 
   \[
   \frac{d}{dx} \int_a^b f(t) \, dt = f(g(x)) \cdot g'(x).
   \]

4. IBP with \( u = \ln x \) and \( dv = x^3 \, dx \).

5. \( u \)-substitution \( u = 4 + x^2 \). Works because of the numerator \( x \). (Trig substitution with \( x = 2 \tan \theta \) would make the problem difficult!)

6. Use the hint. Straightforward PF.

7. 
   \[
   \lim_{a \to 2^+} \int_a^5 \frac{1}{\sqrt{x - 2}} \, dx
   \]
   
   and
   \[
   \int_a^5 \frac{1}{\sqrt{x - 2}} \, dx = 2\sqrt{3} - 2\sqrt{a - 2}.
   \]
   
   Taking the limit as \( a \to 2^+ \), we get \( 2\sqrt{3} \). (Need to correctly state that \( \lim_{a \to 2^+} 2\sqrt{a - 2} = 0 \), but you don’t need to justify (prove) it.)

8. Apply the usual formula to get 
   \[
   \int_0^{\pi/4} \pi \tan x \sec^2 x \, dx = \frac{\pi}{2}.
   \]
   
   Hint: \( \frac{d}{dx} \sec x = \sec x \tan x \). \( u \)-substitution with \( u = \sec x \). Also use hint that \( \sec(\pi/4) = \sqrt{2} \).

9. Under the hypothesis that \( y = e^{kx} \) we have \( \frac{dy}{dx} = ke^{kx} \) and \( \frac{d^2y}{dx^2} = k^2 e^{kx} \). Plug this into the equation to get \( k = ? \) Hint: there are two values of \( k \) that work.

10. Separate variables and integrate. Find the constant of integration from the initial condition \( y(0) = 5 \). Solve for \( y \) as a function of \( x \).