Solutions to selected HW 1 problems

Solution to Problems I, #11. Suppose there is a smallest positive real number, call it \( x \). Then \( 0 < \frac{x}{2} < x \). This contradicts \( x \) being the smallest positive real number.

Solution to Problems I, #17. Let \( P(n) \) be the statement \( 0 < a_n < 5 \).

Base case. Since \( 0 < a_1 = 1 < 5 \), \( P(1) \) is true.

Inductive step. Suppose \( P(k) \) is true for some \( k \in \mathbb{Z}^+ \). Then \( 0 < a_k < 5 \). Clearly

\[
a_{k+1} = \frac{6a_k + 5}{a_k + 2} > 0
\]

since it the quotient of numbers that are positive since \( a_k > 0 \).

Now

\[
\frac{6a_k + 5}{a_k + 2} < 5 \iff 6a_k + 5 < 5(a_k + 2) \quad \text{since} \quad a_k + 2 > 0
\]

\[
\iff 6a_k + 5 < 5a_k + 10
\]

\[
\iff a_k < 5.
\]

Since the inequality \( a_k < 5 \) is true, we conclude that the equivalent inequality \( \frac{6a_k + 5}{a_k + 2} < 5 \) is true. \( \square \)