HW 4. Due date: Friday October 28 at 2 pm (part (a) is revised; in boldface)

1. Chapter 2 Exercise #22 on p. 45. **Hint**: You may use that $\mathbb{Q}$ is dense in $\mathbb{R}$.

2. (a) Describe a sequence $\{a_n\}_{n=1}^{\infty}$ of positive integers whose set of subsequential limits is the set $\mathbb{Z}^+$. 
(b) Let $f : \mathbb{Z}^+ \to \mathbb{Q}$ be a bijection. Define the sequence $\{b_n\}_{n=1}^{\infty}$ by $b_n = f(a_n)$. What is the set of subsequential limits of $\{b_n\}_{n=1}^{\infty}$? **Hint**: Consider Theorem 3.7.
(c) Show that there exists a sequence $\{c_n\}_{n=1}^{\infty}$ of real numbers whose set of subsequential limits is the interval $[0, 1]$.

3. Let $(X, d)$ be a metric space with the property that for every $p \in X$ and $r > 0$, $\bar{B}_r(p) = \{x \in X \mid d(x, p) \leq r\}$ is compact. Let $\{p_n\}_{n=1}^{\infty}$ be a sequence of points in $X$.
(a) Prove that if $\{p_n\}_{n=1}^{\infty}$ has a bounded subsequence, then $\{p_n\}_{n=1}^{\infty}$ has a convergent subsequence.
(b) Choose $q \in X$. Let $N_r = \{n \in \mathbb{Z}^+ \mid d(p_n, q) \leq r\}$. Prove that if there exists $r > 0$ such that $N_r$ is an infinite set, then $\{p_n\}_{n=1}^{\infty}$ has a convergent subsequence.

4. Chapter 2 Exercise #23 on p. 45.
5. Chapter 2 Exercise #24 on p. 45.