In class today, one of the arguments I gave was not correct. It regards the following claim. I used a limit argument which is not complete. Here is a corrected version.

Let \( E = \{x^2 \mid 0 < x < 2\} \subseteq \mathbb{R} \).

You may assume 4 is an upper bound of \( E \).

We want to show that 4 is the least upper bound of \( E \). It suffices to show:

**Claim.** If \( u \in (0, 4) \), then \( u \) is not an upper bound of \( E \).

**Correct proof.** For any \( \epsilon \in (0, 1) \) we have \( 4 - 4\epsilon < 4 - 4\epsilon + \epsilon^2 = (2 - \epsilon)^2 \in E \).

Define \( \epsilon \) so that \( 4 - 4\epsilon = u \). Then \( \epsilon = \frac{4 - u}{4} \in (0, 1) \) and \( u = 4 - 4\epsilon < (2 - \epsilon)^2 \in E \).

Hence \( u \) is not an upper bound of \( E \).