Solving the ODE

$$\frac{dx}{dt} = kx.$$

This is an important ODE since it models the growth of bacteria and decay of radioactive materials to name only a couple of the many phenomena to which this ODE is related. How to solve.

1. Separate variables:

$$\frac{dx}{x} = kdt$$

(Warning: we can only do this under the assumption $x \neq 0$. On the other hand, $x(t) = 0$ is also a solution to the ODE.)

2. Integrate:

$$\ln|x| = kt + \text{Constant}$$

3. Exponentiate:

$$\begin{aligned} |x| &= e^{\text{Constant}} e^{kt} \\ &= \text{PositiveConstant} \cdot e^{kt} \end{aligned}$$

4. Get rid of absolute values:

$$\begin{aligned} x &= \pm\,\text{PositiveConstant} \cdot e^{kt} \\ &= \text{PositiveOrNegativeConstant} \cdot e^{kt} \end{aligned}$$

5. Remember we also have the zero solution $x = 0 = 0 \cdot e^{kt}$.
6. Combining together: the general solution is

$$x = \text{Constant} \cdot e^{kt}$$

where Constant is any real number (positive, negative or zero).