Quiz #1    Math 20E    April 07

Two problems, one on each side of this page. Time allowed: 18+ minutes. Calculators are allowed. No notes or books allowed. 20 points total. Show your work!

1. (§1.2 #18. 10 points) Find the line through (3, 1, −2) that intersects and is perpendicular to the line
   \[x = -1 + t, \quad y = -2 + t, \quad z = -1 + t.\]
   [Book Hint: If \((x_0, y_0, z_0)\) is the point of intersection, find its coordinates.]

   My Hints: (a) Consider the vector from \(\vec{v}\) beginning at \((-1, -2, -1)\) and ending at \((3, 1, -2)\). (b) Project \(\vec{v}\) onto \((1, 1, 1)\) to get a vector \(\vec{p}\). (c) Figure out \((x_0, y_0, z_0)\) in terms of \(\vec{p}\). (d) Describe the line passing through \((3, 1, -2)\) and \((x_0, y_0, z_0)\).

   Answer:
   (a) \(\vec{v} = (4, 3, -1)\).
   (b) \(\vec{p} = \frac{(4, 3, -1) - (1, 1, 1)}{\|(1, 1, 1)\|} = (1, 1, 1) = (2, 2, 2)\).
   (c) \((x_0, y_0, z_0) = (-1, -2, -1) + (2, 2, 2) = (1, 0, 1)\).
   (d) \(\vec{l}(t) = (1, 0, 1) + t(2, 1, -3)\).

2. (§1.3 #16c. 10 points) Find the equation for the plane passing through the points \((2, -1, 3), (0, 0, 5),\) and \((5, 7, -1)\).

   Hint: The equation for a plane passing through a point \((x_0, y_0, z_0)\) and normal to a vector \(\vec{n} = (A, B, C)\) is
   \[Ax + By + Cz + D = 0,\]
   where \(D = -Ax_0 - By_0 - Cz_0\).

   Answer: We have
   \[(2, -1, 3) - (0, 0, 5) = (2, -1, -2),\]
   \[(5, 7, -1) - (0, 0, 5) = (5, 7, -6).\]

   Let
   \[
   \vec{n} = (2, -1, -2) \times (5, 7, -1) = \begin{vmatrix}
   \vec{i} & \vec{j} & \vec{k} \\
   2 & -1 & -2 \\
   5 & 7 & -6 
   \end{vmatrix}
   = 20\vec{i} + 2\vec{j} + 19\vec{k}
   \]

   Then \(D = -20 \cdot 0 - 2 \cdot 0 - 19 \cdot 5 = -95\). The equation for the plane is
   \[20x + 2y + 19z - 95 = 0.\]