Quiz #2 Math 20E April 14

Two problems, one on each side of this page. Time allowed: 18+ minutes. Calculators are allowed. No notes or books allowed. 20 points total. Show your work!

1. (§2.3 #15. 10 points) Find the equation to the tangent plane to \( z = x^2 + 2y^3 \) at \((1,1,3)\).

Answer:

\[
\frac{\partial z}{\partial x} = 2x, \\
\frac{\partial z}{\partial y} = 6y^2. 
\]

At \((1,1,3)\) we have

\[
\frac{\partial z}{\partial x} = 2, \\
\frac{\partial z}{\partial y} = 6. 
\]

Thus the equation to the tangent plane is

\[
z = 3 + 2(x - 1) + 6(y - 1) \\
= 2x + 6y - 5. 
\]

2. (§2.4 #16. 10 points) Determine the equation of the tangent line to the path \((\cos^2 t, 3t - t^3, t)\) at \(t = 0\).

Answer:

\[
\vec{c}(t) = (\cos^2 t, 3t - t^3, t). \\
\vec{c}'(t) = (-2\cos t\sin t, 3 - 3t^2, 1). \\
\vec{c}'(0) = (0, 3, 1). \\
\vec{l}(t) = \vec{c}(0) + t\vec{c}'(0) \\
= (1, 0, 0) + t(0, 3, 1). 
\]