Let $X$ be the set of infinite sequences of 0’s and 1’s. That is,
$$X = \{\{a_n\}_{n=1}^{\infty} | a_n \in \{0, 1\} \text{ for } n \in \mathbb{Z}^+\}.$$

**Proposition.** $X$ is not countable.

**Proof.** Clearly $X$ is not finite. Suppose that $X$ is denumerable. Then there exists a bijection $a : \mathbb{Z}^+ \rightarrow X$. This implies that
$$X = \{a(i) | i \in \mathbb{Z}^+\}.$$

Note that for each $i \in \mathbb{Z}^+$, $a(i)$ is a sequence of 0’s and 1’s.

Define a sequence $b = \{b_n\}_{n=1}^{\infty}$ of 0’s and 1’s as follows:
$$b_n = |a(n) - 1|.$$

That is, if $a(n)_n = 0$, then $b_n = 1$; if $a(n)_n = 1$, then $b_n = 0$. In particular, $b \in X$ and $b_n \neq a(n)_n$ for $n \in \mathbb{Z}^+$. This implies that for each $n \in \mathbb{Z}^+$, the sequence $b = \{b_k\}_{k=1}^{\infty}$ is not equal to the sequence $a(n) = \{a(n)_k\}_{k=1}^{\infty}$. But since $X = \{a(i) | i \in \mathbb{Z}^+\}$, this implies $b \notin X$, a contradiction. □