Math 20D  Lecture A  Midterm 1  Version K
5 Questions; Each problem is worth 10 points; 50 points total.

Instructions:
1. Write your Name, PID, Section, and Exam Version (K, L or M) on the front of your Blue Book.
2. The only things you are allowed to use are writing instruments and erasers and one page, double-sided and handwritten, of notes. (NO calculators, electronic devices, or book.)
3. Write your solutions clearly in your Blue Book and indicate the number and letter of each question.
4. Start each answer on a new page, in the same order they appear in the exam.
5. Show all of your work. No credit will be given for unsupported answers.

1. Solve initial value problem for the linear first order ODE:

\[ y' + 2ty = te^{-\frac{t^2}{4}}, \quad y(0) = -\frac{4}{3}. \]

Include in your answer what the integrating factor \( \mu(t) \) is.

2. Consider the autonomous equation:

\[ y' = (y + 1)^2(y - 1)(y - 3). \]

(a) Find all equilibrium solutions.

(b) Draw a phase line.

(c) Identify each equilibrium solution as stable, unstable, or semistable.

3. Show that the equation is exact and find the general solution:

\[
(2\sin x \cos x + 4xy^3) \, dx + (3e^{\cos y} \sin y + 6x^2y^2) \, dy = 0
\]

4. Solve the initial value problem

\[ y'' - 6y' + 13y = 0, \quad y(0) = -7 \]

Since there is only one initial condition, your answer should have exactly one unknown constant.

5. Use the method of reduction of order to find a second solution of the differential equation:

\[ ty'' + (4 + t)y' + 3y = 0, \quad t > 0; \quad y_1 = \frac{1}{t^3}. \]

Hint: (i) \( \frac{4+t}{t} = \frac{4}{t} + 1 \), (ii) You may use the following formula:

\[
\int t^n e^{at} \, dt = \frac{1}{a} t^n e^{at} - \frac{n}{a} \int t^{n-1} e^{at} \, dt
\]

for any positive integer \( n \) and any nonzero real number \( a \).