#1. Let \( f : [a, b] \to \mathbb{R} \) and let \( v : I \to \mathbb{R} \), where \( I \) is an interval containing \( f([a, b]) \). Suppose that \( f \) is continuous at \( x \in [a, b] \). Suppose that \( \lim_{s \to f(x)} v(s) = v(f(x)) \). Using the \( \varepsilon\)-\( \delta \) definition of limit, prove that \( \lim_{t \to x} v(f(t)) = v(f(x)) \).

#2. Let \( g : I \to \mathbb{R} \) be a function, where \( I \) is an interval containing 0 in its interior. Suppose \( \lim_{x \to 0} \frac{g(x)}{x} = 0 \). Define

\[
 f(x) = \begin{cases} 
 g(x) \sin \frac{1}{x} & \text{if } x \neq 0, \\
 0 & \text{if } x = 0.
\end{cases}
\]

Prove that \( f'(0) = 0 \). HINT: Use the squeeze theorem.

#3. Let \( f, g : \mathbb{R} \to \mathbb{R} \) be differentiable functions satisfying \( f'(x) > g'(x) \) for all \( x \in \mathbb{R} \) and \( f(0) = g(0) \). Prove that \( f(x) > g(x) \) for \( x > 0 \) and \( f(x) < g(x) \) for \( x < 0 \).