Math 140B Midterm 1
5 Questions; 50 points total (10 points each).

Instructions:
1. Write your Name, PID, and section (B01 Th 2 pm or B02 Th 3 pm) on the front of your Blue Book. Copy the following table on the upper right corner of the front of your bluebook:

<table>
<thead>
<tr>
<th>Problem</th>
<th>Score</th>
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2. The only things you are allowed to use are writing instruments and erasers and one page, double-sided and handwritten, of notes. (NO calculators, electronic devices, or book.)

3. Write your solutions clearly in your Blue Book and indicate the number and letter of each question.

4. Start each answer on a new page, in the same order they appear in the exam.

5. Show all of your work. No credit will be given for unsupported answers. The default is to prove results directly, without using major theorems from Rudin.

Notations, definitions, and facts:
\( \mathbb{R} \) denotes the set of real numbers.
\( \mathbb{Z}^+ = \{1, 2, 3, \ldots\} \) denotes the set of positive integers.
sup denotes the supremum, a.k.a., least upper bound.
inf denotes the infimum, a.k.a., greatest lower bound.

\( E \) is dense in \([a, b]\) if every point of \([a, b]\) is a limit point of \( E \) or a point of \( E \), or both.
\( f \) is a real-valued function on \([a, b]\), where \(-\infty < a < b < +\infty\).

\( P \) is a partition \( \{x_i\}_{i=0}^n \) of \([a, b]\), where \( a = x_0 < x_1 < \cdots < x_n = b \).
\( I_i \) is the interval \([x_{i-1}, x_i]\); we call each \( I_i \) a partition interval.
\( \Delta x_i = x_i - x_{i-1} \), the length of \( I_i \).
\(|I| \) denotes the length of an interval, e.g., if \( I_i = [x_{i-1}, x_i] \), then \(|I_i| = \Delta x_i \).
\( M_i = \sup_{x \in I_i} f(x) \) and \( m_i = \inf_{x \in I_i} f(x) \).
\( \alpha \) denotes a nondecreasing function on \([a, b]\).

\( U(P, f) \) and \( L(P, f) \) are the upper and lower Riemann sums, respectively.
\( U(P, f, \alpha) \) and \( L(P, f, \alpha) \) are the upper and lower Riemann–Stieltjes sums, respectively.
\( \mathcal{R} \) is the set of Riemann integrable functions on \([a, b]\).
\( \mathcal{R}(\alpha) \) is the set of Riemann–Stieltjes integrable functions on \([a, b]\), with respect to \( \alpha \).

Riemann–Stieltjes integrable functions are always assumed to be bounded.

\( A - B = A \cap B^c = \{x \in A \mid x \notin B\} \).

A sequence of real numbers \( \{a_n\}_{n=1}^\infty \) is strictly decreasing if \( a_{n+1} < a_n \) for all \( n \geq 1 \).