HOW TO PLAY SU DOKU (unfinished)

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Rules of the game

The board
Here’s what the board looks like. Sort of like a chess or checker board. But it is 9 by 9 instead of 8 by 8. Also note that there are nine 3 by 3 boxes outlined by thick lines.

The end
Perhaps it is easiest to look at a typical end position of Su Doku.

The above diagram is an example of a position we want to end up with. It is a $9 \times 9$ box with all the squares filled with numbers between 1 and 9. Each row of the box has all different numbers. For example, the first row is:
These are the numbers from 1 to 9 in mixed up order. Now every other row and every column is the same way. So each column of the box has all different numbers. They each contain all of the numbers from 1 to 9. Besides this, every $3 \times 3$ box contains all of the numbers from 1 to 9. For example, the $3 \times 3$ box on the upper left corner is:

```
9 8 5
6 2 4
1 7 3
```

**In the beginning**

This is where the game might have started from. In the end diagram above, the numbers in this beginning position where shaded in light gray.

```
  8 5
  6 2
  1 7
  3 6
  7 5
  9 3
  2 4
  9 5

  5
  2
  1
  6
  4
  7
  8

  3
  9
  6
  1
  2
  5
  4
  7
```

At the beginning, we only have partial information of where the numbers are. The game of Su Doku requires us to logically deduce where the rest of the numbers are. Note that there will end up being 9 ones, 9 twos, 9 threes, etc. One for each $3 \times 3$ box, one for each row, and one for each column.

Let’s see how we can figure out what’s in one of the empty squares.
These three 6s tell us there cannot be any 6s in the light shaded squares below. The reason is that if a 6 is in a row, then no other square in that row can have a 6. Similarly, if a 6 is in a column, then no other square in that column can have a 6. Also, if a 6 is in a 3 × 3 box, then no other square in that box can have a 6.

So by the process of elimination, we deduce that there is a 6 in the dark shaded square in the left lower 3 × 3 box above.

Let’s look at a second step. Our position is now:
Elementary techniques

Now that we know how to play Su Doku, we describe the basic techniques that one can use to find the placement of numbers.

**When only one remains**

**Baby steps**

Here’s the easiest situation:

<table>
<thead>
<tr>
<th>4</th>
<th>7</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>6</td>
<td>3</td>
</tr>
</tbody>
</table>

The last number must be different than the rest:

<table>
<thead>
<tr>
<th>4</th>
<th>7</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>8</td>
<td>6</td>
<td>3</td>
</tr>
</tbody>
</table>
Similarly, if:

\[
\begin{array}{ccccccc}
2 & 8 & 3 & 9 & 1 & 7 & 5 & 4 \\
\end{array}
\]

Then:

\[
\begin{array}{ccccccc}
2 & 8 & 3 & 9 & 1 & 7 & 6 & 5 & 4 \\
\end{array}
\]

The simple idea of getting down to the last number has various guises. Suppose we have the following diagram:

\[
\begin{array}{ccc}
7 & & \\
& 7 & \\
& & 2 & 3 \\
\end{array}
\]

Since there is are 7s in the first two rows, the number 7 cannot be in either of the first two rows of the 3 × 3 box on the far right. The only possibility is that the 7 is in the lower left corner:

\[
\begin{array}{ccc}
7 & & \\
& 7 & \\
& & 7 & 2 & 3 \\
\end{array}
\]

Suppose we have four 8s placed as follows:

\[
\begin{array}{ccc}
8 & & \\
& 8 & \\
& & 8 \\
\end{array}
\]
The placement of the 8s determines the placement of the 8 in the middle 3 × 3 box:

![8s placement diagram]

A variant of the above technique is:

![4s placement diagram]

The 4 must go in the middle.

Here’s yet another variation:
And the 2 goes in the dark shaded square.

The elimination method

Here’s another form of the elimination method:

The remaining possibilities for the 3 squares in the column are 3, 6 and 7. But 3 and 6 are in the row, so the only possibility for the shaded square is 7.
The shaded square must have a 5.

**Intermediate techniques**

*Two for the road*

When only two remain in a row:

```
3 4 1 2 5 7 9
```

We have a pair of choices:

```
6 or 8 3 6 or 8 4 1 2 5 7 9
```

We need more information to determine the choice of 6 or 8. Such as the following:
The 8 at the bottom tells us there is a 6 in the shaded square.

The same thing applies to a 3 × 3 box:

```
1 2 3 or 8
4 5 6
7 3 or 8 9
```

And

```
1 2
4 5 6
7 9

---
3
```

Gives us an 8 in the top shaded square and a 3 in the bottom shaded square.

**Square number 1, 2 or 3**

Similarly, suppose we know the following:
Since the 6 has to be in one of the squares on the right or the left, the middle square must contain the 3.

Here’s one way this situation could arise:

The shaded square has a 5. The logic to see this is illustrated by the following:
Likewise

Tells us:

Here are some variations of this.
And a 2 goes in the shaded square.

With a pair of choices we can do the following:

From the information on the top row we know the shaded square has a 1 or a 2. But the information from the 2 in the column tells us it must be a 1.

More of the same:
How did we figure out an 8 is in the light shaded square?

A 9 must be one of the dark shaded squares. So the light shaded square must have an 8.

The following is also a version of the elimination method:
There is only one possible spot for the 4 in the first column:

When a little bit of uncertainty doesn’t matter

To illustrate the idea, here is a simple situation. Suppose we know the following:
Although we don’t know which of the shaded squares has a 6 and which has an 8, we know that the pair 6 and 8 are in the two squares. We conclude that the 7 is in the remaining square:

Here’s an interesting situation:

In the middle $3 \times 3$ box on the bottom:

The 9 can’t be in the top row so it must go in one of the dark shaded squares. So we’re uncertain about the exact location of the 9 but we know which row it is in: the middle one. This means that the 9 in the bottom left $3 \times 3$ box must be on the bottom row:
Combining this with the 9 not being in either of the first two columns, we find where the 9 belongs:

**Advanced techniques**

*When the remaining boxes are pairs of choices – the contradiction method*

Consider the following Su Doku diagram. Most of it has been filled in.
We have the following options. Note that in each undecided box there is exactly one pair of choices.

Here’s a funny quote: ‘When you assume, you make an ass out of u and me.’ As we shall see below, the quote doesn’t apply to Su Doku!

Let’s assume that the number in the upper left box is an 8. Then we can start to fill in the rest of the boxes based on this assumption.
We get a contradiction. Neither a 1 nor a 2 will work in the box below the upper left corner since there is already a 1 and a 2 in the first column. This means our assumption that the number in the upper left box is an 8 is false. Therefore the number in the upper left box is a 6. It is then easy to fill in the remaining numbers.