Bo Li

Weak Convergence Methods for Variational Models of Crystalline Solids

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Preface

This book is based on a set of lecture notes that I first wrote for an advanced graduate course in applied mathematics, entitled *Weak Convergence Methods for Variational Problems Modeling Crystalline Solids*, at the University of California, Los Angeles, in Spring, 1998. These notes were revised later and partially used again, together with other material, for a similar but broader course: *Mathematical and Computational Problems in Materials Science*, at the University of Maryland, College Park, in Fall, 2003.

My intention is to introduce to readers basic concepts and techniques of weak convergence and direct methods in the calculus of variations; and demonstrate how they can be applied to variational models of crystalline solids, in particular, the microstructure of martensite. For years, these concepts and techniques have been proven very powerful in studying many more problems arising in various areas of science.

This book consists of six chapters. Each chapter begins with a brief overview, and ends with exercise problems that provide additional examples and proofs, and other supplemental material.

Chapters 1–3 cover three general and correlated mathematical topics: weak convergence, quasiconvexity, and compensated compactness. In Chapter 1, the concept and theory of weak convergence are reviewed first in abstract Banach and Hilbert spaces, and then in Lebesgue and Soboleve spaces with illustrative examples. In Chapter 2, the notion of quasiconvexity is introduced; and the basic theory of quasiconvexity in the calculus of variations of multiple integrals is presented. As useful tools and methods for studying nonlinear partial differential equations and nonconvex variational problems, Young measures and compensated compactness are discussed in details in Chapter 3.

Chapters 4–6 focus on the description, and the mathematical and numerical treatment of martensitic microstructure. Martensitic microstructure is fine mixture of coherent phases or phase variants of a martensitic crystal, such as a shape-memory alloy; and appears during a martensitic phase transition—a diffusionless, reversible, and structural solid-to-solid phase transition. In Chapter 4, the basic physics of martensite is reviewed and properties of effective free-energy functionals modeling martensitic microstructure are described. In Chapter 5, the tool of Young measures is used to characterize martensitic microstructure. Other related mathematical problems are also studied. Finally, in Chapter 6, the numerical analysis of nonconvex variational problems modeling microstructure is presented.

Readers with a background of the advanced calculus, linear algebra, and the basics of real analysis can understand most of the material. Any knowledge of functional analysis, partial differential equations, and numerical analysis is helpful but not essential. No knowledge of crystalline solids is assumed. For those who wish to learn more about the physical aspects of martensite, I warmly recommend the book by Kaushik Bhattacharya, *Microstructure of Martensite: Why It Forms and How It Gives Rise to the Shape-Memory Effect*, Oxford University Press, 2003.

I am grateful to Richard James for introducing to me the fascinating subject of the microstructure of martensite, to Mitchell Luskin for guiding and supporting me on the related research, and to Vladimir Šverák for teaching me many topics covered in this book. I thank Stuart Antman and Eitan Tadmor for encouraging me to publish these notes. I am in debt to many of the graduate students who took those of my courses at the University of California, Los Angeles, and University of Maryland, College Park, and helped me correct many mistakes and typos in my original notes. Finally, I thank Zelinda Collins of the University of California, San Diego, for transforming my original handwriten notes into Latex files, and Achi Dosanjh of Springer for her patience and exerllent editorial work.

Bo Li
La Jolla, California
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