Math 285J: Seminar in Applied Mathematics, Spring, 1998

Weak Convergence Methods for Variational Problems Modeling Crystalline Solids

MW 4:00 pm – 5:15 pm, MS 5128

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In this course we shall first review the basics of the calculus of multiple integrals and the theory of compensated compactness. We shall then focus on a recently developed mathematical theory of crystalline microstructure based on the principle of energy minimization.

The following is a list of tentative topics to be covered in this course.

- The theory of quasi-convexity: sequential weak lower semi-continuity of multiple integrals; existence theorems in the calculus of variations; partial regularity of minimizers; quasi-convexification. Various concepts of convexity and their relations, characterization of null-Lagrangians.
- The theory of compensated compactness: the Div-Curl lemma and its generalizations; necessary conditions; sufficient conditions; Young measures and gradient Young measures. Some applications in nonlinear partial differential equations.
- Some background of the mechanics of crystalline solids: Bravais lattices; martensitic phase transformation; the Cauchy-Born rule; twinning; Hadamard compatibility condition; classification of interfaces.
- The Ball-James theory of microstructure: the two-well problem; the reduction from a multi-well problem to a two-well one; simple laminates for a six-well problem. Kinematics of various microstructures, restrictions on microstructure.

Most of our references are recently published research articles. But the following books contain some subjects listed above.

- B. Dacorogna, Direct methods in the calculus of variations, Springer-Verlag, 1989.
- L. C. Evans, Weak convergence methods for nonlinear partial differential equations, CBMS Regional conference series in mathematics 74, AMS, 1990.
- P. G. Ciarlet, Mathematical elasticity, Vol. 1: three-dimensional elasticity, North-Holland, 1988.

This course is designed for graduate students who are interested in one of the areas of partial differential equations, the calculus of variations, applied analysis, and mathematical aspects of materials science. A graduate real analysis course will be a prerequisite. But no knowledge on crystalline solids is assumed. The material presented in this course will be essentially self-closed.