ENEE 241/MATH 242, Spring 2002
MIDTERM EXAM 2

Name: ____________ Id: __________ Section: ______ TA: ____________

Problem	1	2	3	4	Total
Score					

Please read the following carefully before you start.

- This is a close-book and close-note exam. No calculators are allowed. There are 4 problems of total 100 points. To get credit, you must show your work.
- There are total 4 pages. On each page, please print your name, student ID number, section number, and your TA's name. Answer each question on the same page. If you need more space, please only use the back of the same page.
- After you finish the exam, please write by hand the following pledge and sign in the box below: *I pledge on my honor that I have not given or received any unauthorized assistance on this examination.*

1. (25 points) (a) How many steps in the bisection method are needed to approximate a zero of a function in $[a, b]$ with the absolute error less than a positive number ϵ?

(b) Use the bisection method to find a root of $f(x) = 1 - x^2 + x^3$ on $[-1, 1]$ with two steps.

(c) Find the first iterate in Newton's method for solving $x^3 - x^2 = 1$ with $x_0 = 1$.

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2. (25 points) (a) Approximate the integral $\int_{-1}^{3} |x|\, dx$ using Simpson's rule with two subintervals.

(b) Write a Matlab script for evaluating the integral $\int_0^{10} e^{-x^2}\, dx$, using the two-point Gaussian quadrature with the number of subintervals $N = 100$.

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3. (25 points) (a) Determine A_1, A_2, and A_3 so that the degree of exactness of the numerical integration rule

$$\int_{-1}^{1} f(x)\,dx \approx A_1 f\left(-\sqrt{\frac{3}{5}}\right) + A_2 f(0) + A_3 f\left(\sqrt{\frac{3}{5}}\right)$$

is as large as possible.

(b) After finding A_1, A_2, and A_3, determine the degree of exactness of this numerical integration rule.

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4. (25 points) Consider the initial-value problem $y' = t + y$ $(0 \le t \le 10)$ and $y(0) = 0$. Let $h = 2$.

(a) Find y_1 by the following Runge-Kutta method

$$y_{n+1} = y_n + hf\left(t_n + \frac{h}{2}, y_n + \frac{h}{2}f(t_n, y_n)\right).$$

(b) Find y_2 by the Adams-Bashforth two-step method

$$y_{n+1} = y_n + \frac{h}{2}[3f(t_n, y_n) - f(t_{n-1}, y_{n-1})]$$

as a predictor and the Adams-Moulton two-step method

$$y_{n+1} = y_n + \frac{h}{12}[5f(t_{n+1}, y_{n+1}) + 8f(t_n, y_n) - f(t_{n-1}, y_{n-1})]$$

as a corrector with one-step correction.

(c) Does y_2 approximate: (i) $y(0.2)$; (ii) $y(2)$; (iii) $y(4)$; or (iv) none of these?