

**ENEE 241/MATH 242, Spring 2002  
MIDTERM EXAM 2**

Name: \_\_\_\_\_ Id: \_\_\_\_\_ Section: \_\_\_\_\_ TA: \_\_\_\_\_

Problem	1	2	3	4	Total
Score					

**Please read the following carefully before you start.**

- This is a close-book and close-note exam. No calculators are allowed. There are 4 problems of total 100 points. To get credit, you must show your work.
- There are total 4 pages. On each page, please print your name, student ID number, section number, and your TA's name. Answer each question on the same page. If you need more space, please only use the back of the same page.
- After you finish the exam, please write by hand the following pledge and sign in the box below: *I pledge on my honor that I have not given or received any unauthorized assistance on this examination.*

1. (25 points) (a) How many steps in the bisection method are needed to approximate a zero of a function in  $[a, b]$  with the absolute error less than a positive number  $\epsilon$ ?  
(b) Use the bisection method to find a root of  $f(x) = 1 - x^2 + x^3$  on  $[-1, 1]$  with two steps.  
(c) Find the first iterate in Newton's method for solving  $x^3 - x^2 = 1$  with  $x_0 = 1$ .

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2. (25 points) (a) Approximate the integral  $\int_{-1}^3 |x| dx$  using Simpson's rule with two subintervals.  
(b) Write a Matlab script for evaluating the integral  $\int_0^{10} e^{-x^2} dx$ , using the two-point Gaussian quadrature with the number of subintervals  $N = 100$ .

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3. (25 points) (a) Determine  $A_1$ ,  $A_2$ , and  $A_3$  so that the degree of exactness of the numerical integration rule

$$\int_{-1}^1 f(x) dx \approx A_1 f\left(-\sqrt{\frac{3}{5}}\right) + A_2 f(0) + A_3 f\left(\sqrt{\frac{3}{5}}\right)$$

is as large as possible.

(b) After finding  $A_1$ ,  $A_2$ , and  $A_3$ , determine the degree of exactness of this numerical integration rule.

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4. (25 points) Consider the initial-value problem  $y' = t + y$  ( $0 \leq t \leq 10$ ) and  $y(0) = 0$ . Let  $h = 2$ .

(a) Find  $y_1$  by the following Runge-Kutta method

$$y_{n+1} = y_n + hf \left( t_n + \frac{h}{2}, y_n + \frac{h}{2} f(t_n, y_n) \right).$$

(b) Find  $y_2$  by the Adams-Bashforth two-step method

$$y_{n+1} = y_n + \frac{h}{2}[3f(t_n, y_n) - f(t_{n-1}, y_{n-1})]$$

as a predictor and the Adams-Moulton two-step method

$$y_{n+1} = y_n + \frac{h}{12}[5f(t_{n+1}, y_{n+1}) + 8f(t_n, y_n) - f(t_{n-1}, y_{n-1})]$$

as a corrector with one-step correction.

(c) Does  $y_2$  approximate: (i)  $y(0.2)$ ; (ii)  $y(2)$ ; (iii)  $y(4)$ ; or (iv) none of these?