

MAPL/CMSC 666-667: Numerical Analysis
Fall semester, 2000
Instructor: Bo Li

A Tentative Course Outline
Note: an * indicates an optional topic.

MAPL/CMSC 666: Numerical Analysis (I)

Chapter 1. Polynomial Approximation (8 lectures)

- (1) Weierstrass Approximation Theorem and Bernstein's Proof
- (2) best uniform approximation: existence, uniqueness, and characterization
- (3) Chebyshev polynomials of first kind
- (4) modulus of continuity, Lipschitz functions, Jackson's Theorems
- (5) weighted least squares approximation, Gram-Schmidt orthogonalization
- (6) orthogonal polynomials: properties and examples

Chapter 2. Polynomial Interpolation (9 lectures)

- (1) Lagrange interpolation, Lagrange formula, remainder, Peano kernel
- (2) Newton formula and divided differences, Hermite-Genocchi formula
- (3) convergence of Lagrange and piecewise Lagrange interpolation polynomials
- (4) Hermite interpolation, divided differences with repeated points
- (5) trigonometric interpolation, fast Fourier transform
- (6) splines and natural splines, spline interpolation, B-splines

Chapter 3. Numerical Quadrature (8 lectures)

- (1) degree of precision, method of undetermined coefficients, basic quadrature and their composite rules
- (2) Peano Kernel Theorem, interpolatory quadrature, Newton-Cotes formulas
- (3) Euler-Maclaurin formula, Richardson extrapolation, Romberg integration
- (4) weighted Gaussian quadrature: formula, error, and convergence; Gauss-Legendre quadrature
- (5) convergence of sequences of integral approximations
- (6) singular integrals, adaptive numerical integration

Chapter 4. Numerical Solution of Systems of Linear Equations (9 lectures)

- (1) Gaussian elimination, backward substitute, partial pivoting, strictly diagonally dominant matrix
- (2) LU factorization, Cholesky factorization, tridiagonal matrix
- (3) vector and matrix norms, spectral radius, error bounds, condition number
- (4) least squares problem, Gram-Schmidt orthogonalization, Householder and Givens QR methods
- (5) basic iterative methods (Jacobi, Gauss-Seidel, relaxation) and their convergence
- (6) conjugate gradient and preconditioned conjugate gradient methods, incomplete Cholesky factorization

Chapter 5. Eigenvalue Problems (8 lectures)

- (1) power and inverse power methods
- (2) *Hessenberg reduction and Hyman's method

- (3) QR algorithm and its convergence, Schur decomposition, shifted QR algorithm
- (4) singular value decomposition, computation of singular values
- (5) tridiagonalization, eigenvalues of tridiagonal Hermitian matrix: QR method, Sturm sequence
- (6) Rayleigh quotient iteration and its convergence
- (7) *eigenvalue perturbation theory

MAPL/CMSC 667: Numerical Analysis (II)

Chapter 6. Numerical Solution of Nonlinear Systems (8 lectures)

- (1) Q-order and R-order of convergence, linear and superlinear convergence
- (2) fixed point iteration and its convergence
- (3) Newton's method: derivation and convergence
- (4) modified Newton methods, Broyden's rank-1 method
- (5) secant method
- (6) finding roots of polynomials
- (7) Sturm sequence of polynomials, bisection method

Chapter 7. Numerical Optimization

- (1) unconstrained minimization, some preliminaries (gradient, Hessian, etc.)
- (2) Newton's method and quasi-Newton's methods
- (3) line search methods: steepest descent methods and conjugate gradient methods
- (4) convergence of conjugate gradient methods
- (5) trust region methods
- (6) simulated annealing methods

Chapter 8. Numerical Solution of Ordinary Differential Equations

- (1) review of ODE theory: existence; uniqueness; stability, model problems and their stability, finite-time blow-up for nonlinear equations, revised existence results, solution to first-order linear equation, general solutions to a linear homogeneous high-order ODE, Gronwall inequality, a lemma
- (2) Euler's method: derivation, truncation error and consistency, convergence and error estimates, numerical stability and rounding errors, asymptotic expansion
- (3) linear multistep methods: some examples, general definition: local discretization error; consistency; convergence; and stability
- (4) linear multistep methods: necessary and sufficient for consistency and for order-m discretization error, example of high order but divergent method, a convergence theorem
- (5) One step methods
- (6) Runge-Kutta methods
- (7) Stiffness

Chapter 9. Numerical Solution of Partial Differential Equations: Finite Difference Methods

- (1) boundary value problems of second elliptic problems, five-point discretization of Laplacian, maximum principle, convergence and error estimate
- (2) initial and boundary value problem for the heat equation, explicit and implicit time discretization, consistency, stability, and convergence

- (3) second order hyperbolic equations, finite difference methods
- (4) hyperbolic equations of conservation laws, upwinding

Chapter 10. Numerical Solution of Partial Differential Equations: Finite Element Methods

- (1) weak formulation, finite elements, interpolation errors
- (2) error in energy norm and L^2 -norm, dual argument