\( \Lambda = \begin{bmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_n \end{bmatrix}, \quad \Lambda^{1/2} = \begin{bmatrix} \sqrt{\lambda_1} & & \\ & \ddots & \\ & & \sqrt{\lambda_n} \end{bmatrix}. \) Then

\[ Rq_i = Q \Lambda^{1/2} Q^T q_i = Q \Lambda^{1/2} e_i = Q \Lambda^{1/2} e_i = \Lambda^{1/2} Q e_i = \Lambda^{1/2} q_i \]

This is what I meant in Problem 4 above, that multiplying the identity out, we see the eigenvectors of \( A \) are precisely the eigenvectors of \( \Lambda A \), and its corresponding eigenvalue is \( \Lambda \). By above if \( \Lambda > 0 \), of course \( \Lambda > 0 \), hence \( R \) is PSD. The last part is a routine verification.

4. Decide whether the following matrices are positive definite, negative definite, semidefinite, or indefinite.

\[
A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 4 \\ 3 & 4 & 9 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & -2 \\ 0 & 0 & -1 \end{bmatrix} \Rightarrow A \text{ is indefinite.}
\]

\[
B = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 2 & 6 & -2 & 0 \\ 0 & -2 & 5 & -2 \\ 0 & 0 & -2 & 3 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 2 & -2 & 0 \\ 0 & 0 & 3 & -2 \\ 0 & 0 & 0 & 5/3 \end{bmatrix} \Rightarrow B \text{ is PSD.}
\]

C is NSD & Pivots of \( -B \) are the negative pivots of \( B \).

D: \( A^{-1} \) is indefinite. Signs of eigenvalues of \( A \) are \((+,+,+)\). Recall eigenvalues of \( A^{-1} \)
are \( 2^{-1} \). Then eigenvalues of \( A^{-1} \) have same sign as eigenvalues of \( A \).

Consider \(-x^2 - 5y^2 - 9z^2 - 4xy - 6xz - 8yz = 1\). This is the quadratic form associated to \( A \).

So this question is \(- (x^T A x) = 1\), or \( x^T A x = -1 \). There is a solution to this? Notice \( A \) is indefinite, hence it is not the case \( x^T A x \geq 0 \) \( \forall x \) i.e. \( \exists x \) s.t. \( x^T A x < 0 \).