Math 110A: Introduction to Partial Differential Equations, Winter 2014

Midterm 2 Review

1. The solution to the initial-value problem of diffusion equation $u_t - ku_{xx} = 0$ $(-\infty < x < \infty, t > 0)$ with $u(x,0) = \phi(x)$ $(-\infty < x < \infty)$ is given by
$$u(x,t) = \int_{-\infty}^{\infty} S(x-y,t)\phi(y)\,dy \quad (-\infty < x < \infty, t > 0),$$
What is the formula of $S(x,t)$? We need this to solve the half-line diffusion problem.

2. Given a function $\phi = \phi(x)$ defined on $x > 0$. How to extend it to be an odd (or even) function on the entire line $-\infty < x < \infty$?

3. Consider the diffusion equation $u_t - ku_{xx} = 0$ on the half line $x > 0$ with the initial condition $u(x,0) = \phi(x)$ $(x > 0)$ and some boundary condition at $x = 0$. Find the solution if the boundary condition is: (1) $u(0,t) = 0$ $(t > 0)$; or (2) $u_x(0,t) = 0$ $(t > 0)$.

4. The solution to the initial-value problem of the wave equation $u_{tt} - c^2 u_{xx} = 0$ $(-\infty < x < \infty, t > 0)$ with $u(x,0) = \phi(x)$ and $u_t(x,0) = \psi(x)$ $(-\infty < x < \infty)$ is given by d'Alembert's formula. What is that formula? What is the domain of dependence? We need this formula for solving the half-line wave equation.

5. Consider the wave equation $u_{tt} - c^2 u_{xx} = 0$ on the half line $x > 0$ with the initial condition $u(x,0) = \phi(x)$ and $u_t(x,0) = \psi(x)$ $(x > 0)$ and some boundary condition at $x = 0$. Find the solution if the boundary condition is: (1) $u(0,t) = 0$ $(t > 0)$; or (2) $u_x(0,t) = 0$ $(t > 0)$.

6. Consider the wave equation $u_{tt} - c^2 u_{xx} = 0$ on a finite interval $(0,l)$ with the initial condition $u(x,0) = \phi(x)$ and $u_t(x,0) = \psi(x)$ $(0 < x < l)$ and the homogeneous Dirichlet or Neumann boundary conditions. Obtain the solution formula using the method of reflection, and find the solution $u = u(x,t)$ at a given point (x,t) using the geometrical construction of reflection.

7. Solve the diffusion equation $u_t - ku_{xx} = f(x,t)$ $(-\infty < x < \infty, t > 0)$ with the initial condition $u(x,0) = \phi(x)$ $(-\infty < x < \infty)$.

8. How to solve the half-line diffusion equation $u_t - ku_{xx} = f(x,t)$ $(x > 0, t > 0)$ with the initial condition $u(x,0) = \phi(x)$ $(x > 0)$ with the boundary condition: (1) $u(0,t) = g(t)$ $(t > 0)$; or (2) $u_x(0,t) = h(t)$ $(t > 0)$?

9. Solve the wave equation $u_{tt} - c^2 u_{xx} = f(x,t)$ on the entire line with the initial condition $u(x,0) = \phi(x)$ and $u_t(x,0) = \psi(x)$.

10. Solve the wave equation $u_{tt} - c^2 u_{xx} = f(x,t)$ on the half-line with the initial condition $u(x,0) = \phi(x)$ and $u_t(x,0) = \psi(x)$ and the boundary condition: (1) $u(0,t) = g(t)$ $(t > 0)$; or (2) $u_x(0,t) = h(t)$ $(t > 0)$.

11. What are the eigenvalues and eigenfunctions for $-X'' = \lambda X$ $(0 < x < l)$ with the boundary conditions: (1) $X(0) = X(l) = 0$? and (2) $X'(0) = X'(l) = 0$?

12. How to solve the initial-boundary-value problem of the diffusion equation $u_t - ku_{xx} = 0$ $(0 < x < l, t > 0)$ with the initial condition $u(x,0) = \phi(x)$ and the boundary conditions: (1) $u(0,t) = 0$ and $u(l,t) = 0$ $(t > 0)$; and (2) $u_x(0,t) = 0$ and $u_x(l,t) = 0$ $(t > 0)$?

13. Use the method of separation of variables to solve the wave equation $u_{tt} - c^2 u_{xx} = 0$ on a finite interval with the initial conditions $u(x,0) = \phi(x)$ and $u_t(x,0) = \psi(x)$ and the boundary conditions (1) $u(0,t) = 0$ and $u(l,t) = 0$ $(t > 0)$; or (2) $u_x(0,t) = 0$ and $u_x(l,t) = 0$ $(t > 0)$.