

Math 130B: Ordinary Differential Equations, Spring 2013
Review for Final Exam

Note. The final exam will cover the following sections:

1. Chapter 7: 7.0, 7.1, 7.2 (exclude Dulac's Criterion), 7.3, 7.4, 7.5, 7.6;
2. Chapter 8: 8.0, 8.1, 8.2, 8.3, 8.4 (exclude scaling laws), 8.5 (exclude homoclinic bifurcation and hysteretic current-voltage curve), 8.7 (exclude Example 8.7.4);
3. Chapter 9: 9.0, 9.2, 9.3;
4. Chapter 10: 10.0, 10.1, 10.2, 10.3, 10.5.

Review

1. Definition of limit cycles. Why linear systems do not have limit cycles?
2. What is a gradient system? Why a gradient system does not have a limit cycle?
3. What is a Liapunov function? Construct such functions for simple cases.
4. What is Poincaré–Bendixson Theorem. The method of trapping region. Use this method to prove the existence of a closed orbit.
5. Liénard systems. Liénard's Theorem. Application of this theorem to the van der Pol equation. Example 7.5.1.
6. Two-timing method and averaged equations for weakly nonlinear oscillators.
7. Two-dimensional saddle-node, transcritical, and (supercritical and subcritical) pitchfork bifurcations: model equations, bifurcation diagrams, and phase portraits.
8. Supercritical and subcritical Hopf bifurcations. Typical equations.
9. Global bifurcations of cycles: Hopf bifurcations, saddle-node bifurcation of cycles, infinite-period bifurcation, and homoclinic bifurcation. Phase portraits.
10. Understand the analysis presented on pages 266–270 on the driven pendulum system.
11. Poincaré maps: definition. What is the relation between the existence of a fixed point of a Poincaré map and the existence of a closed orbit?
12. Linearized Poincaré maps and the linear stability of a closed orbit.
13. Basic properties of Lorenz equations: nonlinearity, symmetry, and volume contraction. The formula (2) on page 313 for the rate of change of a volume in phase space.
14. Lorenz equations: Fixed points and their linear stabilities; Global stability of the origin
15. Working definition of chaos. Working definition of strange attractors. Definition of “sensitive dependence on initial data”. Example 9.3.3.
16. One-dimensional maps: fixed points, and their linear stability and superstability. Construction of cobwebs.
17. The logistic map: definition, fixed points, and their linear stabilities.
18. Basic properties of the logistic map. What is period-doubling? How to prove the existence and stability of 2-cycle periodic solutions? 3-cycle?
19. The Liapunov exponent: the definition and the meaning. Calculation of the Liapunov exponent for some systems.