Math 130B: Ordinary Differential Equations and Dynamical Systems
Spring quarter, 2019

Midterm Review

1. **Equilibrium points and their stabilities.**
   (a) Concept of equilibrium points, and their stability, asymptotic stability, and instability.
   (b) Linearizing a nonlinear system around an equilibrium point. Use the The Linearization Theorem to classify equilibrium points.
   (c) Concept of a Liapunov function associated with an equilibrium point. Construct such a function for a simple system. The Liapunov Stability Theorem.
   (d) Nullclines of a planar nonlinear system. Use nullclines to partition the phase plane into disjoint regions and determine qualitatively the vector field in each of these regions.

2. **Bifurcation.**
   (a) Saddle-node, transcritical, and pitchfork bifurcations: model equations, bifurcation diagrams, and phase portraits.
   (b) Hopf bifurcations.

3. **Gradient systems.**
   (a) Concept of a gradient system. A necessary condition of such a system.
   (b) Equilibrium points of a gradient system $\dot{x} = -\nabla V(x)$ are exactly critical points of the potential V. Stabilities of such points in relation to local minima, local maxima, and saddle points of V.
   (c) Prove that the potential V along any trajectory of non-constant solution to the gradient system $\dot{x} = -\nabla V(x)$ strictly decreases with time t.
   (d) Why a gradient system does not permit a non-constant period solution?
   (e) Why trajectories of solutions to a gradient system $\dot{x} = -\nabla V(x)$ are orthogonal to the level surfaces of the potential function V?

4. **Planar Hamiltonian systems.**
   (a) Concept of a planar Hamiltonian system.
   (b) Show that the Hamiltonian on any trajectory is a constant: $\frac{d}{dt}H(x(t), y(t)) = 0$.
   (c) Why any solution $(x(t), y(t))$ to a planar Hamiltonian system will stay in a level curve of the Hamiltonian?

5. **Limit sets and closed orbits.**
   (a) What is the α-limit set of point x? What is the ω-limit set of point x?
   (b) What is a local section at a point?
   (c) How to construct a flow box in a neighborhood of a point?