Math 130B: ODE and Dynamical Systems, Spring 2019

Homework Assignment 1

Due Monday, April 8, 2019

1. Problem 1 of Exercises of Chapter 8: (ii), (iii), and (v).

2. Use a computer program to plot phase portraits of the following systems:
   (i) (van der Pol oscillator) $\dot{x} = y$, $\dot{y} = -x + y(1 - x^2)$.
   (ii) (Dipole fixed point) $\dot{x} = 2xy$, $\dot{y} = y^2 - x^2$.

3. Problem 2 of Exercises of Chapter 8.

4. (Polar coordinates) Use the identity $\theta = \tan^{-1}(y/x)$ to show that $\dot{\theta} = (x\dot{y} - y\dot{x})/r^2$.

5. Consider the system $\dot{x} = y$, $\dot{y} = -x + (1 - x^2 - y^2)y$.
   (i) Let D be the disk $x^2 + y^2 < 4$. Verify that the system satisfies the hypotheses of the existence and uniqueness theorem throughout the domain D.
   (ii) By substitution, show that $x(t) = \sin t$, $y(t) = \cos t$ is an exact solution of the system.
   (iii) Now consider a different solution, in this case starting from the initial condition $x(0) = 1/2$, $y(0) = 0$. Without doing any calculations, explain why this solution must satisfy $x(t)^2 + y(t)^2 < 1$ for all $t < \infty$.

6. Let $\phi : \mathbb{R} \times \mathbb{R}^n \to \mathbb{R}^n$ be a smooth dynamical system. Define

$$F(x) = \left.\frac{d}{dt}\right|_{t=0} \phi(t, x) \qquad \forall x \in \mathbb{R}^n.$$

   Show that, for any $x_0 \in \mathbb{R}^n$, the function $x(t) = \phi(t, x_0)$ solves the initial-value problem

$$\dot{x} = F(x) \qquad \text{and} \qquad x(0) = x_0.$$