

Math 130B: ODE and Dynamical Systems, Spring 2019

Homework Assignment 2

Due Monday, April 15, 2019

1. For each of the following systems: (1) Find all the fixed points and the corresponding linearized systems; (2) Use the linearization theorem to classify these fixed points and determine their stabilities; and (3) Plot the phase portrait:
 - (i) $\dot{x} = x - y, \dot{y} = x^2 - 4$.
 - (ii) $\dot{x} = xy - 1, \dot{y} = x - y^3$.
 - (iii) $\dot{x} = y^2 - 3x + 2, \dot{y} = x^2 - y^2$.
2. Solve the system $\dot{x} = -x, \dot{y} = y + x^2$ to show that the stable and unstable manifolds are given by $S : y = -x^2/3$ and $U : x = 0$.
3. Solve the system $\dot{x} = -x, \dot{y} = -y + x^2, \dot{z} = z + x^2$ to find its stable and unstable manifolds.
4. Problem 5 of Exercises of Chapter 8 (page 185).
5. Problem 9 of Exercises of Chapter 8 (page 185).
6. Consider the system $\dot{x} = y - 2x, \dot{y} = \mu + x^2 - y$.
 - (i) Sketch the nullclines.
 - (ii) Find and classify the bifurcations that occur as μ varies.
 - (iii) Sketch the phase portrait for selected values of μ to illustrate the bifurcations.