Homework Assignment 3
Due Monday, April 22, 2019

1. Consider the equation $\dot{x} = 1 + rx + x^2$, where $r \in \mathbb{R}$ is a parameter.

   (1) Find the critical value $r = r_c$ at which a saddle-node bifurcation occurs.
   (2) Sketch the birfaction diagram of the fixed point x vs. r.

2. Consider the equation $\dot{x} = x - rx(1 - x)$, where $r \in \mathbb{R}$ is a parameter.

   (1) Find the critical value $r = r_c$ at which a transcritical bifurcation occurs.
   (2) Sketch the birfaction diagram of the fixed point x vs. r.

3. Consider the equation $\dot{x} = x + rx/(1 + x^2)$, where $r \in \mathbb{R}$ is a parameter.

   (1) Find the critical value $r = r_c$ at which a pitchfork bifurcation occurs.
   (2) Sketch the birfaction diagram of the fixed point x vs. r.

4. Show that the system $\dot{x} = y + \mu x$, $\dot{y} = -x + \mu y - x^2 y$ undergoes a (supercritical) Hopf bifurcation at $\mu = 0$.

5. Problem 2 of Excieses of Chapter 9 (page 210).

6. Consider the system $\dot{x} = -2y + yz$, $\dot{y} = x - xz$, $\dot{z} = xy$.

   (1) Show that the fixed point $(0, 0, 0)$ is not hyperbolic.
   (2) Find a Liapunov function to show that $(0, 0, 0)$ is stable. (Hint: Try $V(x, y, z) = ax^2 + by^2 + cz^2$.)