Math 130B: ODE and Dynamical Systems, Spring 2019

Homework Assignment 4

Due Monday, April 29, 2019

1. Problem 6 of Exercises of Chapter 9 (page 211).
2. Problem 7 (a) (c) of Exercises of Chapter 9 (page 211).
3. Let X_0 be a strict local minimum of a C^1 function $V = V(X)$. Show that X_0 is an equilibrium point and that the function $V(X) - V(X_0)$ is a strict Liapunov function for the gradient system $\dot{X} = -\operatorname{grad} V(X)$ at X_0.
4. Problem 8 (c) (d) of Exercises of Chapter 9 (page 211).
5. Find the Hamiltonian $H = H(x, y)$ for the system $\dot{x} = y$, $\dot{y} = -x + x^2$ and sketch the phase portrait for this system.
6. Prove that the linearization at an equilibrium point of a planar Hamiltonian system has eigenvalues either $\pm\lambda$ or $\pm\lambda i$ for some real number λ.
7. Problem 16 of Exercises of Chapter 9 (page 212).
8. (Optional) Prove that the flow associated with a planar Hamiltonian system preserves the area. (The general result, true for any n-dimensional Hamiltonian system, is known as Liouville's theorem in statistical mechanics.)