Math 130B: ODE and Dynamical Systems, Spring 2019

Homework Assignment 5

Due Monday, May 13, 2019

1. Identify all the points that lie in either an ω-limit set or an α-limit set of a solution trajectory of the system $\dot{r} = r^3 - 3r^2 + 2r$, $\dot{\theta} = 1$.

2. The Flow Box Theorem (two-dimensional version): In a sufficiently small neighborhood of a non-equilibrium point of the system $\dot{x} = f(x, y)$, $\dot{y} = g(x, y)$, where f and g are C^1-functions, there is a differentiable change of coordinates $u = u(x, y)$, $v = v(x, y)$ such that $\dot{u} = 0$ and $\dot{v} = 1$.

   Show that the nonlinear change of coordinates $u = x + y^3$, $v = y + y^2$ satisfies the requirement of the flow box theorem for the system

   $$\dot{x} = -\frac{3y^2}{1 + 2y}, \quad \dot{y} = \frac{1}{1 + 2y}$$

   in the neighborhood of any point (x, y) with $y \neq 1/2$.

3. Consider the three-dimensional system in the cylindrical coordinates $\dot{r} = r(1 - r)$, $\dot{\theta} = 1$, $\dot{z} = -z$. Compute the Poincaré map $P : S \to S$ along the closed orbit $r = 1$, $z = 0$, where $S = \{(\theta, r, z) : \theta = 0, r > 0, z \in \mathbb{R}\}$, and show that this closed orbit is asymptotically stable.

4. Consider the three-dimensional system in the cylindrical coordinates $\dot{r} = r(1 - r)$, $\dot{\theta} = 1$, $\dot{z} = z$. Compute again the Poincaré map $P : S \to S$ along the closed orbit $r = 1$, $z = 0$. where $S = \{(\theta, r, z) : \theta = 0, r > 0, z \in \mathbb{R}\}$, What can you now say about the behavior of solutions near the closed orbit? Sketch the phase portrait for this system.

5. The system $\dot{r} = r(1 - r^2)$, $\dot{\theta} = 1$ has a closed orbit $r = 1$. Compute the Poincaré map defined on the positive x-axis for this orbit and determine its stability.