1. Use Dulac’s criterion to show that the system \( \dot{x} = y, \dot{y} = -x - y + x^2 + y^2 \) has no closed orbits. (Hint: try \( g(x, y) = e^{-2x} \).)

2. Consider the system \( \dot{x} = x - y - x(x^2 + 2y^2), \dot{y} = x + y - y(x^2 + 2y^2) \).
   (1) Write the system in the polar coordinates. (You can use \( rr' = x\dot{x} + y\dot{y} \) and \( \dot{\theta} = (\dot{y}x - \dot{x}y)/r^2. \)
   (2) Use the trapping region method to show that this system has a closed orbit in the region defined by \( r_1 < r < r_2 \) for some positive numbers \( r_1 \) and \( r_2 \) with \( 0 < r_1 < r_2 \).

3. Show that the system \( \dot{x} = x - y - x^3, \dot{y} = x + y - y^3 \) has a periodic solution.

4. Consider the two-dimensional system \( \dot{X} = AX - ||X||^2X, \) where \( A \) is a \( 2 \times 2 \) constant real matrix with complex eigenvalues \( \alpha \pm i\beta \) (\( \alpha, \beta \in \mathbb{R} \) and \( \beta \neq 0 \)). Prove that there exists at least one limit cycle if \( \alpha > 0 \) and that there are none if \( \alpha < 0 \).

5. Use the Liénard theorem to show that the van der Pol equation \( \ddot{x} + \mu(x^2 - 1)\dot{x} + x = 0 \) has a unique stable limit cycle for any parameter \( \mu > 0 \).