**Math 130B: ODE and Dynamical Systems, Spring 2019**

# Homework Assignment 8

**Due Friday, June 7, 2019**

1. Find all the fixed points for the one-dimensional map $x_{n+1} = f(x_n)$ and determine their stabilities, where the function $f(x)$ is given by:

   (1) $f(x) = x^2$;
   (2) $f(x) = 3x - x^3$.

2. Show that the map $x_{n+1} = f(x_n)$ with $f(x) = 1 + (1/2)\sin x$ has a unique fixed point $x^*$ that is globally stable (i.e., $x_n \to x^*$ for any $x_0$).

3. Show that the iteration $x_{n+1} = \cos x_n$ has a unique fixed point. Predict the stability of this fixed point by drawing the web diagram. Can you prove your statement?

4. Find possible periodic points with period $n = 1$ or 2 for each of the following maps and classify them as attracting, repelling, or neither.

   (1) $f(x) = x - x^2$;
   (2) $f(x) = 2(x - x^2)$.

5. Discuss the bifurcations that occur in the following families of maps at the indicated parameter value:

   (1) $S_\lambda(x) = \lambda \sin x$ at $\lambda = 1$;
   (2) $E_\lambda(x) = \lambda e^x$ at $\lambda = 1/e$.

6. Assume that $x_0$ lies on a cycle of period $n$ for the map defined by $f(x)$. Show that

$$(f^n)'(x_0) = \prod_{i=0}^{n-1} f'(x_i),$$

   where $x_i = f(x_{i-1})$ $(i = 1, \ldots, n)$. Conclude that

$$(f^n)'(x_0) = (f^n)'(x_j), \qquad j = 1, \ldots, n-1.$$