**Math 140C: Foundations of Real Analysis, Spring 2012**

# Midterm Review

## Chapter 8. Some Special Functions

1. Concept of power series and that of analytic functions. Radius of convergence. How to find the radius of convergence of a power series? The formula: $c_k = f^{(k)}(0)/k!$ $(k = 0, 1, 2, \dots)$.
2. Proof of Theorem 8.2 .
3. Theorem 8.3: $\sum_{i=1}^{\infty} \sum_{j=1}^{\infty} a_{ij} = \sum_{j=1}^{\infty} \sum_{i=1}^{\infty} a_{ij}$. What are the assumptions?
4. Theorem 8.4: from $f(x) = \sum_{n=0}^{\infty} c_n x^n$ to $f(x) = \sum_{n=0}^{\infty} b_n (x-a)^n$. How the intervals of convergence of these two power series are related?
5. True or false: if a power series $f(x) = \sum_{n=1}^{\infty} c_n x^n$ vanishes on an open interval, then all $c_n = 0$.
6. How the function $e^x$ is defined using power series? How $\log x$ is defined? Basic definition of $\sin x$ and $\cos x$ using power series.
7. Concept of an orthonormal series.
8. Theorem 8.11 and its proof.
9. Theorem 8.12 and its proof.
10. Trigonometric series. Calculations of coefficients. The Dirichlet kernel: definition and properties (cf. page 189).
11. Theorem 8.14 and its proof.
12. The statement of Parseval's Theorem.
13. Definition of the Gamma function. Theorem 8.18 and its proof. Stirling's formula.

## Chapter 9. Functions of Several Variables

1. Basic concept of linear combination, span, linear dependence, linear independence, basis, and dimension.
2. Linear transformations. The definition of $\|A\|$ for any $A \in L(\mathbb{R}^n, \mathbb{R}^m)$.
3. The definition and matrix representation of total derivative. Relation of total derivative and partial derivatives. Theorem 9.17.