Monday, June 12, 2017 10:16 PM

6.1

2. $\vec{w} = \begin{bmatrix} 3 \\ -1 \\ -5 \end{bmatrix}$ $\vec{x} = \begin{bmatrix} 6 \\ -2 \\ 3 \end{bmatrix}$

$\vec{w} \cdot \vec{w} = 3^2 + (-1)^2 + (-5)^2$
$= 35$

$\vec{w} \cdot \vec{x} = \vec{x} \cdot \vec{w} = (6 \cdot 3) + (2 \cdot 1) + (-5 \cdot 3)$
$= 18 + 2 - 15$
$= 5$

$\frac{\vec{x} \cdot \vec{w}}{\vec{w} \cdot \vec{w}} = \frac{5}{35} = \frac{1}{7}$

7. $\|\vec{w}\| = \sqrt{\vec{w} \cdot \vec{w}} = \sqrt{35}$

10. $\vec{v} = \begin{bmatrix} -6 \\ 4 \\ -3 \end{bmatrix}$

$\|\vec{v}\| = \sqrt{6^2 + 4^2 + 3^2}$
$= \sqrt{61}$

$\hat{v} = \frac{1}{\sqrt{61}} \begin{bmatrix} -6 \\ 4 \\ -3 \end{bmatrix}$

15. $\vec{a} = \begin{bmatrix} 8 \\ -5 \end{bmatrix}$ $\vec{b} = \begin{bmatrix} -2 \\ -3 \end{bmatrix}$

$\vec{a} \cdot \vec{b} = -16 + 15$
$= -1$

19. a) True. By definition
b) True. By definition
c) True:
$\|\vec{u} - \vec{v}\| = \|\vec{u}\| + \|\vec{v}\| - 2\vec{u} \cdot \vec{v}$
$\|\vec{u} - (-\vec{v})\| = \|\vec{u} + \vec{v}\| = \|\vec{u}\| + \|\vec{v}\| + 2\vec{u} \cdot \vec{v}$
If $\vec{u}$ & $\vec{v}$ are $\perp$, $\vec{u} \cdot \vec{v} = 0$,
so $\|\vec{u} - \vec{v}\| = \|\vec{u} + \vec{v}\|$
d) False:
Let $A = \begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix}$
null A has the vector $\vec{u} = [1, 1]^T$
because $A\vec{u} = 0$
col A has the vector $\vec{v} = [1, 0]^T$

$\vec{u} \cdot \vec{v} = 1$, so $\vec{u}$ and $\vec{v}$ are not $\perp$

e) True:

$\vec{w} = c_1\vec{v}_1 + c_2\vec{v}_2 + \cdots c_p\vec{v}_p$; $\vec{w} \in W$

$\vec{x}\cdot\vec{v}_1 = \vec{x}\cdot\vec{v}_2 = \cdots x\cdot\vec{v}_p$ implies that

$\vec{x}\cdot\vec{w} = \underbrace{0 + 0 + \cdots 0}_{p\text{-times}}$

$\vec{x}\cdot\vec{w} = 0$ for all possible $\vec{w}$ in W

which means that $\vec{x} \in W^T$

22. $\vec{u} = (u_1, u_2, u_3)$

$\vec{u}\cdot\vec{u} \geq 0$ because $\vec{u}\cdot\vec{u} = u_1^2 + u_2^2 + u_3^2$

and u_1^2, u_2^2, and u_3^2 are all ≥ 0.

The sum of positive numbers is also positive.

$\vec{u}\cdot\vec{u} = 0$ iff $\vec{u} = \vec{0}$

24. $\|\vec{u}+\vec{v}\|^2 + \|\vec{u}-\vec{v}\|^2$ (look at Prob. 19c))

$= \|\vec{u}\|^2 + \|\vec{v}\|^2 + \cancel{2\vec{u}\cdot\vec{v}} + \|u\|^2 + \|\vec{v}\| - \cancel{2\vec{u}\cdot\vec{v}}$

$= 2\|\vec{u}\|^2 + 2\|\vec{v}\|$

29. Exact argument as Prob. 19e)