2. \( \vec{w} = \begin{bmatrix} 3 \\ -1 \\ -5 \end{bmatrix}, \quad \vec{x} = \begin{bmatrix} 6 \\ -2 \\ 3 \end{bmatrix} \)

\[ \vec{w} \cdot \vec{w} = 3^2 + (-1)^2 + (-5)^2 = 35 \]

\[ \vec{w} \cdot \vec{x} = \vec{x} \cdot \vec{w} = (6 \cdot 3) + (2 \cdot 1 + (-5 \cdot 3) = 18 + 2 - 15 = 5 \]

\[ \frac{\vec{x} \cdot \vec{w}}{\vec{w} \cdot \vec{w}} = \frac{5}{35} = \frac{1}{7} \]

7. \[ ||\vec{w}|| = \sqrt{\vec{w} \cdot \vec{w}} = \sqrt{35} \]

10. \[ \vec{v} = \begin{bmatrix} -6 \\ 4 \\ -3 \end{bmatrix} \]

\[ ||\vec{v}|| = \sqrt{(-6)^2 + 4^2 + (-3)^2} = \sqrt{61} \]

\[ \hat{\vec{v}} = \frac{1}{\sqrt{61}} \begin{bmatrix} -6 \\ 4 \\ -3 \end{bmatrix} \]

15. \[ \vec{a} = \begin{bmatrix} 8 \\ -5 \end{bmatrix}, \quad \vec{b} = \begin{bmatrix} -2 \\ -3 \end{bmatrix} \]

\[ \vec{a} \cdot \vec{b} = -16 + 15 = -1 \]

19. a) True. By definition
b) True. By definition
c) True:
\[ ||\vec{u} - \vec{v}|| = ||\vec{u}|| + ||\vec{v}|| - 2 \vec{u} \cdot \vec{v} \]
\[ ||\vec{u} - (-\vec{v})|| = ||\vec{u} + \vec{v}|| = ||\vec{u}|| + ||\vec{v}|| + 2 \vec{u} \cdot \vec{v} \]
If \( \vec{u} \neq \vec{v} \) are \( \perp \), \( \vec{u} \cdot \vec{v} = 0 \), so \[ ||\vec{u} - \vec{v}|| = ||\vec{u} + \vec{v}|| \]
d) False:
Let \( \vec{A} = \begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix} \)
null \( A \) has the vector \( \vec{u} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}^T \)
because \( A\vec{u} = \vec{0} \)
col \( A \) has the vector \( \vec{v} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}^T \)
\( \vec{u} \cdot \vec{v} = 1 \), so \( \vec{u} \) and \( \vec{v} \) are not \( \perp \)
e) True:  
\[ \vec{u} = C_1 \vec{v}_1 + C_2 \vec{v}_2 + \cdots + C_p \vec{v}_p; \quad \vec{v}_i \in \textbf{W} \]

\[ \vec{x} \cdot \vec{v}_1 = \vec{x} \cdot \vec{v}_2 = \cdots = \vec{x} \cdot \vec{v}_p \text{ implies that} \]
\[ \vec{x} \cdot \vec{w} = 0 + 0 + \cdots + 0 \]
\[ \text{p-times} \]
\[ \vec{x} \cdot \vec{w} = 0 \text{ for all possible } \vec{w} \text{ in } \textbf{W} \]
which means that \[ \vec{x} \in \textbf{W}^\perp \]

22. \[ \vec{u} = (u_1, u_2, u_3) \]

\[ \vec{u} \cdot \vec{u} > 0 \text{ because } \vec{u} \cdot \vec{u} = u_1^2 + u_2^2 + u_3^2 \]
and \[ u_1, u_2, \text{ and } u_3 \] are all \( > 0 \). The sum of positive numbers is also positive.
\[ \vec{u} \cdot \vec{u} = 0 \text{ iff } \vec{u} = \vec{0} \]

24. \[ ||\vec{u} + \vec{v}||^2 + ||\vec{u} - \vec{v}||^2 \text{ (look at Prob. 19e)} \]
\[ = ||\vec{u}||^2 + ||\vec{v}||^2 + 2\vec{u} \cdot \vec{v} + ||\vec{u}||^2 + ||\vec{v}||^2 - 2\vec{u} \cdot \vec{v} \]
\[ = 2||\vec{u}||^2 + 2||\vec{v}||^2 \]

29. Exact argument as Prob. 19e)