Math 20D, Lecture A00: Introduction to Differential Equations, Spring 2013

Review for Midterm Exam 1

Chapter 1. Introduction

1. Derivation of Equation (5) on page 3.
2. Meaning of a direction field associated with a first-order differential equation $y' = f(t)$.
3. Concept of initial-value problems and initial conditions. Concept of solutions. Are functions $y_1(t) = t/3$, $y_2(t) = e^{-t} + t/3$, and $y_3(t) = e^{-t}$ all solutions to $y'''' + 4y''' + 3y = t$?
4. Concept of order of an equation, linear and nonlinear equations. Determine if an equation is linear or nonlinear. Is $y'' + yy' = t$ linear? What about $y'' - (y^2 - 1)y' + y = 0$ and $y' = \sin y$?

Chapter 2. First Order Differential Equations

1. Method of integrating factor for solving the first order linear equation $y' + p(t)y = g(t)$.
2. Method for solving separable equations. Initial-value problems. See, e.g., Example 2 on page 45.
3. Example 1 on page 52.
4. Find the solution to $y' = y^2$ and $y(0) = y_0$ where y_0 is a given, nonzero number. Find out the largest interval on which the solution is defined. (You need to consider $y_0 > 0$ and $y_0 < 0$.)
5. What does Theorem 2.4.2 (on page 70) mean?
6. Important: Determine the qualitative behavior of solutions to autonomous equation $y' = f(y)$. Steps: (1) Find critical points defined by $f(x) = 0$ and the corresponding equilibrium solutions; (2) Determine the stability of these equilibrium solutions using the graph of $f(x)$; and (3) Determine monotonicity and (convexity if possible) of solutions with initial values different from these critical points.
7. Exact equations. $M(x, y) + N(x, y)dy/dx = 0$ is exact if $\partial_y M(x, y) = \partial_x N(x, y)$. Method for solving exact equations.

Chapter 3. Second Order Linear Equations

1. Important: Method of solving $ay'' + by' + cy = 0$ with a, b, c being constants. Characteristic equation: $ar^2 + br + c = 0$. Let $\Delta = b^2 - 4ac$. Three cases:
   (1) $\Delta > 0$. Two distinct real roots r_1 and r_2. The general solution is $y = c_1 e^{r_1 t} + c_2 e^{r_2 t}$;
   (2) $\Delta = 0$. Two repeated real roots $r_1 = r_2$. The general solution is $y = (c_1 + c_2 t)e^{r_1 t}$;
   (3) $\Delta < 0$. Two conjugate complex roots $r_1 = \lambda + \mu i$ and $r_2 = \lambda - \mu i$ with λ and μ real numbers and $\mu \neq 0$. The general solution is $y = e^{\lambda t}(c_1 \cos(\mu t) + i \sin(\mu t))$.
2. The domain of a solution to a second order linear equation. See Theorem 3.2.1 on page 146. See also Example 1 on page 147.
3. What is the Principle of Superposition? Where we use that principle?
4. The definition of Wronskians. A formula of the Wronskian (cf. Theorem 3.2.6 on page 153). What is the consequence of the Wronskian of $y_1(t)$ and $y_2(t)$ being nonzero?
5. Solving $ay'' + by' + cy = g(t)$ (with a, b, c constants) for some types of functions $g(t)$: exponential, sin, cos, exponential times trig functions, polynomials, etc.