

Math 210A: Mathematical Methods for Physical Sciences and Engineering, Fall 2017
Review

Chapter 1. Sequences and Series

1. Definition of convergence of a sequence of numbers. Cauchy sequences. Theorem: A Cauchy sequence of real or complex numbers converges. Definition of bounded sequences and monotonic sequences. Theorem: Bounded monotonic sequences converge. Theorem: Any bounded sequence of numbers has a convergent subsequence. The squeeze theorem.
2. Partial sums and convergence of infinite series of numbers. That $\sum_{n=1}^{\infty} a_n$ converges implies that $a_n \rightarrow 0$ as $n \rightarrow \infty$ and $\sum_{n=N}^{\infty} a_n \rightarrow 0$ as $N \rightarrow \infty$. Geometrical series.
3. Absolute and conditional convergence. Comparison test for positive series. Ratio test and root test. Integral test. When does $\sum_{n=1}^{\infty} 1/n^p$ converge? Convergence of alternating series. Definition and convergence of the product of two series.
4. Definition and test of the convergence of infinite product.
5. Different types of convergence of a sequence of functions: pointwise, uniform, $L^2(a, b)$, and weak convergence in $L^2(a, b)$. Statement of Weierstrass Theorem.
6. The Cauchy–Schwarz inequality for functions in $L^2(a, b)$.
7. What is the Heaviside function? What is the Dirac δ function (at the origin)? Approximation of δ function by “regular functions”.
8. Series of functions: different types of convergence. Radius of convergence of a power series. Term-by-term differentiation and integration of power series.
9. Techniques of computing infinite sums using integration or differentiation of a known series of functions.

Chapter 2. Matrix Techniques

1. Definition and properties of determinant. How to compute the determinant of a matrix? Determinant of a diagonal matrix and upper or lower triangle matrix. $A^{-1} = (1/\det A)C^T$ where C is the co-factor matrix of A . How to compute the derivative of a matrix-valued function? The Wronskian. Rank of a matrix. Rank-one matrix.
2. How to find eigenvalues and eigenvectors of a matrix? Characteristic polynomial of a matrix. Matrix diagonalization. Similar matrices.
3. SPD (symmetric positive definite) matrices: definition and three equivalent conditions. How to determine a matrix is SPD? What is a Hessian matrix of a multi-variable function? Spectrum decomposition of a real symmetric matrix.
4. Definition and properties of orthogonal matrices. The Gram–Schmidt orthogonalization. Least-squares problem and solution.
5. Schur’s lemma: triangularization. Singular value decomposition. Polar decomposition.
6. Definition and some basic properties of matrix exponentials. Compute matrix exponential by definition or diagonalization.

Chapter 3. Hilbert Spaces

1. Definition and examples of Hilbert spaces. Law of parallelogram. The Cauchy–Schwarz inequality. Norm. The triangle inequality. Orthogonality. Pythagorean theorem. Convergence. Cauchy sequences. Weak convergence.
2. Best approximation of a vector in a closed subspace of a Hilbert space. How to find such a best approximation? Orthogonality. Gram matrix.
3. Orthogonal systems and polynomials. Complete orthonormal system in a Hilbert space.
4. Legendre polynomials: definition and basic properties.