Math 210B: Mathematical Methods for Physical Sciences and Engineering, Winter 2018

Review for Midterm Exam

Chapter 1. Analytic Functions

1. Complex numbers, real and imaginary parts, module, argument. The nth roots of the unity.
2. Complex functions, limits and continuity: equivalence with the real and imaginary parts.
3. Derivative of a complex function. Formulas of calculating derivatives. The Cauchy–Riemann equations. Concept: analytic in a domain, analytic at a point. Examples of complex functions that are continuous everywhere and nowhere differentiable. $f(z) = |z|$, $\mathrm{Re}\, z$, etc.
4. Elementary functions: e^z, $\sin z$, $\cos z$, $\tan z$, $\log z$, and z^α. Is $\sin z$ a bounded function? Complex functions are mappings: pre-images (i.e., domains) and images (i.e., ranges). Concept of branch.

Chapter 2. Complex Integration

1. Definition of a smooth arc (or curve) and its parametrization, contours, simple contours, loops (i.e., closed contours), orientations.
2. Definition of contour integration. Some basic properties (e.g., linearity, bounds, etc.).

$$\int_\gamma f(z)\, dz = \int_a^b f(z(t))z'(t)\, dt = F(z_{\mathrm{T}}) - F(z_{\mathrm{I}}) \qquad \text{with } F'(z) = f(z).$$

3. Path independence. Deformation Invariance Theorem. Cauchy's Integration Theorem.
4. Simply connected domains. Cauchy's Integration Formula. Analyticity implies the infinite-differentiability.
5. Liouville's Theorem. Mean-Value Theorem. Maximum Principle. Applications of these theorems.

Chapter 3. Series Representations

1. Power series, radius of convergence. Term-by-term limit, integration, and differentiation.
2. Taylor and Maclaurin series of analytic functions and the coefficients. Maclaurin series of some elementary functions.
3. Laurent series, coefficients. Term-by-term limit, integration, and differentiation.
4. Definition of zeros and their multiplicities or orders for an analytic functions in a domain. Simple zeros. If z_0 is a zero of f of order $m(\geq 1)$, then $f(z) = (z - z_0)^m g(z)$ with $g(z_0) \neq 0$. True or fals: If $f(z)$ is analytic in $|z| < 1$ and $f(z) = 0$ for all z with $z = x \in \mathbb{R}$ and $0 < x < 1$, then $f = 0$ in $|z| < 1$.
5. Singularities: removable, pole of order m, and essential. A pole of order m really means that $f(z) = g(z)/(z - z_0)^m$ for some g with $g(z_0) \neq 0$.

Chapter 4. Residue Theory

1. Definition of residue. Calculation of residue by Laurent series. Calculation of the residue of a removable singularity and that of a pole of order m.
2. Cauchy's Residue Theorem and applications to evaluation of integrals.
3. Techniques of integration using the residue theorem, different types of integrals. (Practice!)

Chapter 5. Conformal Mappings

1. Harmonic functions and its invariance.
2. Definition of a conformal mapping. Conformal if analyticity with nonzero derivative.
3. Riemann Mapping Theorem.
4. Möbius transforms, their constructions.