1. Find the order of the pole 0 of the function \( f(z) = (2 \cos z - 2 + z^2)^{-2} \).

2. Prove that if \( f(z) \) has a pole of order \( m \) at \( z_0 \) then \( g(z) := f'(z)/f(z) \) has a simple pole at \( z_0 \). What is the coefficient of \((z - z_0)^{-1}\) in the Laurent expansion for \( g(z) \)?

3. Let \( P(z) \) and \( Q(z) \) be analytic at \( z_0 \). Assume that \( P(z_0) \neq 0 \) and that \( z_0 \) is a simple zero of \( Q(z) \). Show that \( z_0 \) is a pole of \( f(z) := P(z)/Q(z) \) of order 1 and that \( \text{Res}(f; z_0) = \frac{P(z_0)}{Q'(z_0)} \).

4. Determine all the isolated singularities of each of the following functions and compute the residue at each singularity:
   
   \[
   \begin{align*}
   (1) & \quad \frac{z + 1}{z^2 - 3z + 2}; \\
   (2) & \quad \frac{\sin z}{(z - \pi)^2}; \\
   (3) & \quad \frac{e^z}{z(z + 1)^3}.
   \end{align*}
   \]

5. Evaluate the integral
   \[
   \int_C e^{1/z} \sin \left( \frac{1}{z} \right) \, dz,
   \]
   where \( C \) is the simple and positively oriented circle \( |z| = 1 \).

6. Evaluate each of the following integrals where the loop is simple and positively oriented:
   
   \[
   \begin{align*}
   (1) & \quad \int_{|z|=4} \frac{\sin z}{z^2 - 4} \, dz; \\
   (2) & \quad \int_{|z|=3} \frac{e^z}{z^2(z - 2)(z + 5i)} \, dz; \\
   (3) & \quad \int_{|z|=1} \frac{1}{z^2 \sin z} \, dz.
   \end{align*}
   \]

7. Using the method of residues to calculate the integral
   \[
   \int_0^\pi \frac{d\theta}{5 + 2 \cos \theta}.
   \]

8. Verify that
   \[
   \int_0^\infty \frac{x^2 + 1}{x^4 + 1} \, dx = \frac{\pi}{\sqrt{2}}.
   \]

9. Calculate
   \[
   \text{p.v.} \int_{-\infty}^\infty \frac{dx}{(x^2 + 1)(x^4 + 1)}.
   \]

10. Calculate
    \[
    \int_0^\infty \frac{\cos x}{(x^2 + 1)^2} \, dx.
    \]