Math 210B: Mathematical Methods in Physical Sciences and Engineering
Winter quarter, 2018

Homework Assignment 4

Due Wednesday, February 7, 2018

1. Find the order of the pole 0 of the function $f(z) = (2\cos z - 2 + z^2)^{-2}$.

2. Prove that if $f(z)$ has a pole of order m at z_0 then $g(z) := f'(z)/f(z)$ has a simple pole at z_0. What is the coefficiect of $(z - z_0)^{-1}$ in the Laurent expansion for $g(z)$?

3. Let $P(z)$ and $Q(z)$ be analytic at z_0. Assume that $P(z_0) \neq 0$ and that z_0 is a simple zero of $Q(z)$. Show that z_0 is a pole of $f(z) := P(z)/Q(z)$ of order 1 and that $\operatorname{Res}(f; z_0) = P(z_0)/Q'(z_0)$.

4. Determine all the isolated singularities of each of the following functions and compute the residue at each singularity:

$$(1) \quad \frac{z+1}{z^2 - 3z + 2}; \qquad (2) \quad \frac{\sin z}{(z-\pi)^2}; \qquad (3) \quad \frac{e^z}{z(z+1)^3}.$$

5. Evaluate the integral

$$\int_C e^{1/z} \sin\left(\frac{1}{z}\right) dz,$$

where C is the simple and positively oriented circle $|z| = 1$.

6. Evaluate each of the following integrals where the loop is simple and positively oriented:

$$(1) \quad \int_{|z|=4} \frac{\sin z}{z^2 - 4}\, dz; \qquad (2) \quad \int_{|z|=3} \frac{e^{iz}}{z^2(z-2)(z+5i)}\, dz; \qquad (3) \quad \int_{|z|=1} \frac{1}{z^2 \sin z}\, dz.$$

7. Using the method of residues to calculate the integral

$$\int_0^\pi \frac{d\theta}{5 + 2\cos\theta}.$$

8. Verify that

$$\int_0^\infty \frac{x^2+1}{x^4+1}\, dx = \frac{\pi}{\sqrt{2}}.$$

9. Calculate

$$\text{p.v.} \int_{-\infty}^\infty \frac{dx}{(x^2+1)(x^4+1)}.$$

10. Calculate

$$\int_0^\infty \frac{\cos x}{(x^2+1)^2}\, dx.$$