1. Let $\rho$ be a positive number and $\rho \neq 1$. Show that the mapping $w = z + 1/z$ maps the circle $|z| = \rho$ onto the ellipse
\[
\frac{u^2}{(\rho + \frac{1}{\rho})^2} + \frac{v^2}{(\rho - \frac{1}{\rho})^2} = 1.
\]

2. Describe the image of each of the following domains under the mapping $w = e^z$:
   (1) The strip $0 < \text{Im} \, z < \pi$;
   (2) The half-strip $\text{Re} \, z < 0$, $0 < \text{Im} \, z < \pi$;
   (3) The rectangle $1 < \text{Re} \, z < 2$, $0 < \text{Im} \, z < \pi$;
   (4) The slanted strip between the two lines $y = x$ and $y = x + 2\pi$.

3. Prove that if $f$ has a simple pole at $z_0$, then there exists a punctured neighborhood at $z_0$, $0 < |z - z_0| < R$ for some $R > 0$, on which $f$ is one-to-one.

4. Find a linear transformation mapping the circle $|z| = 1$ onto the circle $|w - 5| = 3$ and taking the point $z = i$ to $w = 2$.

5. Find a Möbius transformation mapping the lower half-plane to the disk $|w + 1| < 1$.

6. Find a Möbius transformation mapping the unit disk $|z| < 1$ onto the right half-plane and taking $z = -i$ to the origin.

7. Let $w = f(z)$ be a Möbius transformation mapping the points 0, $\lambda$, $\infty$ to $-i$, 1, $i$, respectively, where $\lambda$ is real. For what values of $\lambda$ is the upper half-plane mapped onto $|w| < 1$?

8. Let $w = f(z)$ be a Möbius transformation such that $f(1) = \infty$ and $f$ maps the imaginary axis onto the unit circle $|w| = 1$. What is the value of $f(-1)$?