Homework Assignment 1

Due Monday, April 9, 2018

1. For each of the followings partial differential equations (PDEs), find out its order and determine if it is a linear equation:

    (1) $u_x + uu_y = 0$ with $u = u(x, y)$;
    (2) $u_t - \Delta u = u(u - 1/2)(u - 1)$ with $u = u(x, t)$ and $x \in \mathbb{R}^n$;
    (3) $\Delta u = \sin |x|$ with $u = u(x)$ and $x \in \mathbb{R}^n$;
    (4) $\Delta u - \sinh u = 0$ with $u = u(x, y, z)$;
    (5) $u_{tt} - \Delta u + \Delta^2 u = 0$ with $u = u(x, t)$ and $x \in \mathbb{R}^n$.

2. Let $u \in C^2(\mathbb{R}^n)$. Prove that $\nabla \cdot \nabla u = \Delta u$.

3. Let $\Omega = (0, 1) \times (0, 1)$. Show that there exists no $u \in C^2(\overline{\Omega})$ such that $\Delta u = 1$ in Ω and $u = 0$ on $\partial\Omega$.

4. Verify that $u_n(x, y) = \sin(nx)\sinh(ny)$ is a solution to $\Delta u = 0$ for any $n > 0$.

5. Let $u = u(x, y)$. Solve $u_x x + u = 0$.

6. Let $u = u(x, y)$ and solve the equation $3u_y + u_{xy} = 0$.

7. Let $u = u(x, y)$. Find the general solution to the equation $u_x + 2xy^2 u_y = 0$.

8. Let $u = u(x, y)$. Solve the equation $\sqrt{1 - x^2} u_x + u_y = 0$ with $u(0, y) = e^{-y^2}$.