Math 210C: Mathematical Methods in Physical Sciences and Engineering
Spring quarter, 2018

Homework Assignment 2

Due Wednesday, April 18, 2018

1. Consider the initial-value problem

$$\begin{cases} u_x + y u_y = 0 & \text{for } x, y \in \mathbb{R}, \\ u(x,0) = \phi(x) & \text{for } x \in \mathbb{R}. \end{cases}$$

   Prove the following:

   (1) If $\phi(x) = x$ $(x \in \mathbb{R})$ then this initial-value problem does not have a solution;
   (2) If $\phi(x) = 1$ $(x \in \mathbb{R})$ then this initial-value problem has many solutions.

2. Solve the following Cauchy problems of first-order equations:

   (1) $u_x + x u_y - u_z = u$ and $u(x, y, 1) = x + y$;

   (2) $u u_x + y u_y = x$ and $u(x, 1) = 2x$.

3. Classify each of the following second-order equations into elliptic, hyperbolic, or parabolic equations:

   (1) $2u_{xx} - 6u_{xy} + 3u_{yy} = 0$;
   (2) $u_{xx} - 4u_{xy} + 4u_{yy} = 0$;
   (3) $2u_{xx} - u_{xy} + 3u_{yy} = 0$.

4. Find infinitely many solutions of the form $u(x, y) = f(x) + g(y)$ to the boundary-value problem: $\Delta u = 0$ on the upper half plane $\{(x, y) : y > 0\}$ and the boundary condition $u(x, 0) = 1$ $(-\infty < x < \infty)$.

5. Let a and b be two positive numbers with $a < b$. Solve Poisson's equation $\Delta u = 1$ in $a < r < b$ with the boundary condition $u = 0$ on the circles $r = a$ and $r = b$, where $r = \sqrt{x^2 + y^2}$.

6. Solve the one-dimensional eigenvalue problem $-u'' = \lambda u$ on $(0, L)$ and $u'(0) = u'(L) = 0$ for some given $L > 0$.

7. Use the method of separation of variables to solve the following boundary-value problem of Laplace's equation:

$$\begin{cases} u_{xx} + u_{yy} = 0 & \text{for } 0 < x < \pi, 0 < y < \pi, \\ u_x(0, y) = 0 \text{ and } u_x(\pi, y) = 0 & \text{for } 0 < y < \pi, \\ u(x, 0) = 0 \text{ and } u(x, \pi) = g(x) & \text{for } 0 < x < \pi, \end{cases}$$

   where $g(x)$ is a given, continuous function on $[0, \pi]$.