Math 210C: Mathematical Methods in Physical Sciences and Engineering
Spring quarter, 2018

Homework Assignment 7
Due Friday, June 8, 2018

1. Use Fourier series to solve the initial-value problem for the Klein–Gordon equation

$$\begin{cases} u_{tt} - c^2 u_{xx} + m^2 u = 0 & \text{for } 0 < x < \pi,\ t > 0, \\ u(0,t) = 0 \quad \text{and} \quad u(\pi, t) = 0 & \text{for } t > 0, \\ u(x,0) = g(x) \quad \text{and} \quad u_t(x,0) = h(x) & \text{for } x > 0, \end{cases}$$

where c and m are positive numbers, and g and h are continuous functions on $[0, \pi]$.

2. Find the solution to the initial-value problem of wave-equation

$$\begin{cases} u_{tt} - u_{xx} = 0 & \text{for } x \in \mathbb{R}, t > 0, \\ u(x,0) = \ln(1 + x^2) \quad \text{and} \quad u_t(x,0) = 1 - x & \text{for } x \in \mathbb{R}. \end{cases}$$

3. Solve the initial-value problem

$$\begin{cases} u_{xx} - 3u_{xt} - 4u_{tt} = 0 & \text{for } x, t \in \mathbb{R}, \\ u(x,0) = x^2 \quad \text{and} \quad u_t(x,0) = e^x & \text{for } x \in \mathbb{R}. \end{cases}$$

4. Solve $u_{tt} = u_{xx} + xt$ $(-\infty < x < \infty, t > 0)$ with $u(x,0) = 0$ and $u_t(x,0) = 1 + x$ $(-\infty < x < \infty)$.

5. Solve the initial-value problem of wave equation

$$\begin{cases} u_{tt} - \Delta u = 0 & \text{for } x, y, z \in \mathbb{R},\ t > 0, \\ u(x,y,z,0) = x^2 + y^2 \quad \text{and} \quad u_t(x,y,z,0) = 0 & \text{for } x, y, z \in \mathbb{R}. \end{cases}$$

6. The small wave motion $u = u(x,t)$ of a flexible beam with clamped ends at $x = 0$ and $x = 1$ is the solution to the following initial-boundary-value problem:

$$\begin{cases} u_{tt} + \gamma^2 u_{xxxx} = 0 & \text{for } 0 < x < 1, t > 0, \\ u(0,t) = u(1,t) = 0 \quad \text{and} \quad u_x(0,t) = u_x(1,t) = 0 & \text{for } t > 0, \\ u(x,0) = g(x) \quad \text{and} \quad u_t(x,0) = h(x) & \text{for } 0 < x < 1, \end{cases}$$

where g and h are smooth and bounded functions on $[0, 1]$. Define the energy

$$E(t) = \frac{1}{2} \int_0^1 \left(u_t^2 + \gamma^2 u_{xx}^2 \right) dx.$$

Prove that $E(t)$ is a constant for $t \geq 0$, i.e., the energy is conserved.