1. Let \((X, \mathcal{M}, \mu)\) be a measure space with \(\mu(X) = 1\). Let \(n \in \mathbb{N}\) and \(A_j \in \mathcal{M}\) \((j = 1, \ldots, n)\) be such that \(\sum_{j=1}^{n} \mu(A_j) > n - 1\). Prove that \(\mu(\bigcap_{j=1}^{n} A_j) > 0\).

2. Let \((X, \mathcal{M}, \mu)\) be a measure space. Assume \(E_j \in \mathcal{M}\) \((j = 1, 2, \ldots)\) and \(\sum_{j=1}^{\infty} \mu(E_j) < \infty\). Let \(E = \{x \in X : x \in E_j\ \text{for infinitely many } j\}\). Prove that \(E \in \mathcal{M}\) and \(\mu(E) = 0\).

3. Is it true that \(m(G) = m(G)\) for any open subset \(G \subseteq \mathbb{R}\)? If yes, prove it. If no, provide a counter example.

4. Let \(A\) and \(B\) be two Lebesgue measurable subsets of \(\mathbb{R}\) such that \(m(A) = m(B) < \infty\). Suppose \(E \subseteq \mathbb{R}\) satisfies \(A \subseteq E \subseteq B\). Prove that \(E\) is also Lebesgue measurable and \(m(E) = m(A)\).

5. Let \(\mu\) be the Borel measure on \(\mathbb{R}\) defined by the nondecreasing and right-continuous function

\[
F(x) = \begin{cases} 
0 & \text{if } x \leq 0, \\
x & \text{if } 0 < x < 1, \\
x^2 + 3 & \text{if } 1 \leq x \leq 2, \\
7 & \text{if } 2 < x.
\end{cases}
\]

Calculate: (1) \(\mu((-\infty, 1))\); (2) \(\mu((-\infty, 1])\); (3) \(\mu(\mathbb{R})\); and (4) \(\mu\{2\}\).

6. The Dirac measure \(\delta\) concentrated on \(\{0\}\) is a Borel measure on \(\mathbb{R}\). Find all the increasing and right-continuous functions \(F : \mathbb{R} \to \mathbb{R}\) such that \(\mu_F = \delta\).

7. Prove Proposition 1.20 in the textbook.

8. Let \(0 < \alpha < 1\). Construct a closed set \(F \subseteq [0, 1]\) such that it is nowhere dense and \(m(F) = \alpha\).

9. Let \(E \subseteq \mathbb{R}\) be Lebesgue measurable with \(0 < m(E) < \infty\). Then for any \(\alpha \in (0, 1)\) there exists an open interval \(I\) such that \(m(E \cap I) > \alpha m(I)\).

10. Let \(E \subseteq \mathbb{R}\) be a Lebesgue measurable set and \(N\) the nonmeasurable set described in §1.1.

   (1) If \(E \subseteq N\), then \(m(E) = 0\).

   (2) If \(m(E) > 0\), then \(E\) contains a nonmeasurable set. (It suffices to assume \(E \subseteq [0, 1]\).

In the notation of §1.1, \(E = \bigcup_{r \in \mathbb{Q} \cap [0, 1]} E \cap N_r\).)