1. Let \( X \) be a finite-dimensional normed vector space. Prove the following:
   (1) \( X \) is complete;
   (2) Any two norms on \( X \) are equivalent.
   (See Exercise 6 on page 155.)

2. Let \( \alpha \in (0, 1] \). Denote by \( C^{0,\alpha}([0, 1]) \) the vector space of all real-valued, Hölder continuous functions on \([0, 1]\) of exponent \( \alpha : f \in C^{0,\alpha}([0, 1]) \) means that \( f \in C([0, 1]) \) and
   \[
   [f]_\alpha := \sup_{x, y \in [0, 1], x \neq y} \frac{|f(x) - f(y)|}{|x - y|^\alpha} < \infty.
   \]
   Prove that
   \[
   \|f\| = \sup_{x \in [0, 1]} |f(x)| + [f]_\alpha
   \]
   is a norm on \( C^{0,\alpha}([0, 1]) \), and with this norm, \( C^{0,\alpha}([0, 1]) \) is a Banach space.

3. Let \( X \) be a normed vector space and \( M \) a closed subspace of \( X \). Prove the following:
   (1) \( \|x + M\| = \inf\{\|x + y\| : y \in M\} \) is a norm on the quotient space \( X/M \);
   (2) For any \( \varepsilon > 0 \) there exists \( x \in X \) such that \( \|x\| = 1 \) and \( \|x + M\| \geq 1 - \varepsilon \);
   (3) The projection map \( \pi(x) = x + M \) from \( X \) to \( X/M \) has norm 1;
   (4) If \( X \) is complete, so is \( X/M \).

4. Let \( X \) be an infinite-dimensional normed vector space. Prove the following:
   (1) There is a sequence \( \{x_j\} \) in \( X \) such that \( \|x_j\| = 1 \) for all \( j \) and \( \|x_j - x_k\| \geq 1/2 \) for \( j \neq k \).
   (2) \( X \) is not locally compact.
   (See Exercise 19 on page 160.)

5. If \( X \) is a Banach space and \( X^* \) is separable, then \( X \) is separable. (See Exercise 25 on page 160.)