1. Let $\mu$ be a Radon measure on a locally compact Hausdorff space $X$. Let $\phi \in C(X, (0, \infty))$.
Define
$$\nu(E) = \int_E \phi \, d\mu \quad \forall E \in \mathcal{B}_X.$$ 
Define also
$$I(f) = \int f \phi \, d\mu \quad \forall f \in C_c(X).$$
Clearly $I : C_c(X) \to \mathbb{C}$ is linear and positive. Let $\nu'$ be the Radon measure associated with $I$. Prove the following:
(1) If $U$ is an open subset of $X$, then $\nu(U) = \nu'(U)$;
(2) The Borel measure $\nu$ is outer regular on all Borel sets;
(3) $\nu = \nu'$ and hence $\nu$ is a Radon measure.
(See Exercise 9 on page 220 for some hints.)

2. Let $\mu$ be a Radon measure on a locally compact Hausdorff space $X$ such that $\mu(\{x\}) = 0$ for any $x \in X$. Assume $A \in \mathcal{B}_X$ such that $0 < \mu(A) < \infty$. Prove that for any $\alpha \in \mathbb{R}$ with $0 < \alpha < \mu(A)$, there exists $B \in \mathcal{B}_X$ such that $B \subseteq A$ and $\mu(B) = \alpha$.

3. Let $\mu$ be a Radon measure on a locally compact Hausdorff space $X$. Assume $f \in L^1(\mu)$ is real-valued. Let $\varepsilon > 0$. Prove that there exist a lower semi-continuous function $g$ on $X$ and an upper semi-continuous function $h$ on $X$ such that
$$h \leq f \leq g \quad \text{on } X \quad \text{and} \quad \int_X (g - h) \, d\mu < \varepsilon.$$

4. Prove that the Banach space $C([0, 1])$ is not reflexive.

5. Let $X$ be a locally compact Hausdorff space and $f, f_n \in C_0(X)$ ($n = 1, 2, \ldots$). Prove that $f_n \to f$ weakly in $C_0(X)$ if and only sup$_{n \geq 1} \|f_n\| < \infty$ and $f_n(x) \to f(x)$ for all $x \in X$.

6. Let $\mu_n = (1/n) \sum_{k=1}^n \delta_{k/n}$ ($n = 1, 2, \ldots$), where $\delta_a$ denotes the Dirac measure concentrated at $a \in \mathbb{R}$. Let $m$ denote the Lebesgue measure. Denote
$$\langle f, \mu \rangle = \int_X f \, d\mu$$
for any Random measure $\mu$ on $[0, 1]$ and any $f \in C([0, 1])$.
(1) Prove that $\langle f, \mu_n \rangle \to \langle f, m \rangle$ for any $f \in C([0, 1])$.
(2) Is it true that $\mu_n(E) \to m(E)$ for any Borel set $E \subseteq [0, 1]$?