1. Let $X$ and $Y$ be locally compact Hausdorff spaces. Let $\mu$ and $\nu$ be Radon measures on $X$ and $Y$, respectively. Assume $f \in C_c(X \times Y)$. Prove that the functions

$$x \mapsto \int_Y f_x(y) d\nu(y) \quad \text{and} \quad y \mapsto \int_X f_y(x) d\mu(x)$$

are continuous functions on $X$ and $Y$, respectively.

2. Let $X$ and $Y$ be locally compact Hausdorff spaces. Let $\mu$ and $\nu$ be Radon measures on $X$ and $Y$, respectively. (They are not necessary $\sigma$-finite.) Assume $f : X \times Y \to \mathbb{R}$ is nonnegative lower semi-continuous. Prove that the functions

$$x \mapsto \int_Y f_x(y) d\nu(y) \quad \text{and} \quad y \mapsto \int_X f_y(x) d\mu(x)$$

are Borel-measurable functions on $X$ and $Y$, respectively, and

$$\int \int f \, d(\hat{\mu} \times \nu) = \int \int f \, d\mu \, d\nu = \int \int f \, d\nu \, d\mu.$$