Math 270C: Numerical Mathematics
Spring quarter, 2007
Homework Assignment 1
Due Tuesday, April 10

1. Let $B_n f \in \mathcal{P}_n$ $(n = 0, 1, \cdots)$ be the Bernstein polynomials of $f \in C[0,1]$.
   (a) Let $f_0(x) = 1$, $f_1(x) = 1$, and $f_2(x) = x^2$. Show that
   $$B_n f_0(x) = 1, \quad B_n f_1(x) = x, \quad B_n f_2(x) = \frac{n-1}{n} x^2 + \frac{1}{n} x, \qquad \forall x \in [0,1].$$
   (b) In general, is $B_n f \in \mathcal{P}_n$ the best uniform approximation of $f \in C[0,1]$ in $\mathcal{P}_n$ on $[0,1]$?
2. Let $f \in C[a,b]$ but $f \notin \mathcal{P}$. Show that there exits no polynomial $p \in \mathcal{P}$ such that
   $$\|f - p\|_{C[a,b]} \le \|f - q\|_{C[a,b]} \qquad \forall q \in \mathcal{P}.$$
3. Let $x_0, \cdots, x_n$ be $n+1$ distinct points in $[a,b]$. Let $f \in C[a,b]$. Does there exist a unique $p \in \mathcal{P}_n$ such that
   $$\max_{0 \le j \le n} |f(x_j) - p(x_j)| \le \max_{0 \le j \le n} |f(x_j) - q(x_j)| \qquad \forall q \in \mathcal{P}_n?$$
4. Let $f(x) = x^4$ $(0 \le x \le 1)$. Find the best uniform approximation of f in $\mathcal{P}_1$ on $[0,1]$.