

# Math 270C: Numerical Mathematics

Spring quarter, 2007

## Homework Assignment 6

Due Tuesday, May 29

1. Suppose a sequence of nonnegative numbers  $\{e_i\}_{i=0}^n$  satisfy

$$e_i \leq (1 + \delta)e_{i-1} + B, \quad i = 1, \dots, n,$$

for some constants  $\delta > 0$  and  $B \geq 0$ . Show that

$$e_i \leq e^{i\delta} e_0 + \left( \frac{e^{n\delta} - 1}{\delta} \right) B, \quad i = 0, \dots, n.$$

2. Consider the initial-value problem

$$y' = x - x^2, \quad y(0) = 0.$$

Suppose we use Euler's method with stepsize  $h$  to compute approximate values  $y_k$  for  $y(x_k)$ , where  $x_k = kh$  ( $k = 0, \dots$ ).

(a) Calculate explicitly the error  $y(x_k) - y_k$  for  $k \geq 0$ .

(b) Fix  $x > 0$ . Let  $h = x/N$ . Prove directly that  $e(x, h) := y(x) - y_N \rightarrow 0$  as  $N \rightarrow \infty$ .

3. Consider the initial-value problem

$$y' = \sqrt{y}, \quad y(0) = 0.$$

Let  $x > 0$ . Let  $N \geq 1$  be an integer. Define  $h = x/N$  and  $x_k = kh$ ,  $k = 0, \dots, N$ . Use Euler's method to find  $y_N$  that approximates  $y(x_N) = y(x)$ . Does Euler's method work in this case? Explain why.