Math 10C
Final Examination
June 11, 2012
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Version A

Instructions

1. You may use any type of calculator, but no other electronic devices during this exam.
2. You may use one page of notes, but no books or other assistance during this exam.
3. Write your *Name*, *PID*, and *Section* on the front of your Blue Book.
4. Write the *Version* of your exam on the front of your Blue Book.
5. Write your solutions clearly in your Blue Book
    (a) Carefully indicate the number and letter of each question and question part.
    (b) Present your answers in the same order they appear in the exam.
    (c) Start each question on a new side of a page.
6. Read each question carefully, and answer each question completely.
7. Show all of your work; no credit will be given for unsupported answers.

1. (6 points) The company Colonel Electric has produced a new brand of light bulb. Suppose that x measures the number of hours elapsed before one of these light bulbs fails. The *cumulative density function* for x is given by

$$P(x) = \begin{cases} 0 & \text{if } x < 0, \\ 1 - e^{-200x} & \text{if } x \geq 0. \end{cases}$$

    (a) What is the probability that a light bulb lasts for more than 50 hours?
    (b) What is the median value of the number of hours these light bulbs last?

2. (6 points) Let $\vec{u} = -\vec{i} + 2\vec{j} - 4\vec{k}$ and $\vec{v} = \vec{j} + \vec{k}$.

    (a) Find $\vec{u} \cdot \vec{v}$.
    (b) Find $\vec{u} \times \vec{v}$.
    (c) Find $\vec{u} \cdot (\vec{u} \times \vec{v})$.

3. (6 points)

    (a) Find an equation for the plane tangent to the graph of $f(x, y) = \ln(5x + 6y)$ at the point $(-1, 1, 0)$.
    (b) Find a linear approximation for $f(-0.9, 0.8)$.

Exam continues on other side.

4. (6 points) An ant is marching on a metal plate whose temperate at (x, y) is $3x^2y - y^3$ degrees Celsius. When the ant is at the point $(1, 2)$, it is anxious to move in the direction in which the temperature *drops* the most rapidly.

   (a) Find the unit vector in the direction in which the temperature drops most rapidly at the instant the ant departs $(1, 2)$.

   (b) If the ant mistakenly moves toward the point $(0, 1)$, what rate of change will it experience at the instant it departs $(1, 2)$?

5. (6 points) Let $W(s, t) = F(u(s, t), v(s, t))$, where $u(1, 0) = 2$, $u_s(1, 0) = -1$, $u_t(1, 0) = 5$, $v(1, 0) = 3$, $v_s(1, 0) = 5$, $v_t(1, 0) = 4$, $F_u(2, 3) = -1$, and $F_v(2, 3) = 10$.

   (a) Find $W_s(1, 0)$.

   (b) Find $W_t(1, 0)$.

6. (6 points) Consider the function

$$f(x, y) = \frac{x^4}{4} - \frac{x^2}{2} + y^2.$$

   (a) Find the critical points of $f(x, y)$.

   (b) For each critical point, classify it as a local maximum, minimum, or saddle point.

7. (6 points) A rectangular box with length x, width y and height z has a volume of 5 cm^3. The four sides of the box are made of a material that costs $\$10/$cm^2, while the top and bottom each cost $\$50/$cm^2.

   (a) Find the dimensions of the box that minimize the total cost of the box.

   (b) For the dimensions found in part (7a), compute the cost of the box.

8. (6 points) Use Lagrange multipliers to find the maximum and minimum values of $f(x, y) = x^2 + 2y^2$ subject to the constraint $x^2 + y^2 = 1$.