# §1.1: 2, 5, 8, 16, 18, 23, 24, 26, 28.

$\boxed{2}$

$$3(133, -0.33, 0) + (-399, 0.99, 0) =$$
$$(3\cdot 133, 3\cdot(-0.33), 0) + (-399, 0.99, 0) =$$
$$(399, -0.99, 0) + (-399, 0.99, 0) =$$
$$(399 - 399, -0.99 + 0.99, 0) =$$
$$\boxed{(0, 0, 0)}$$

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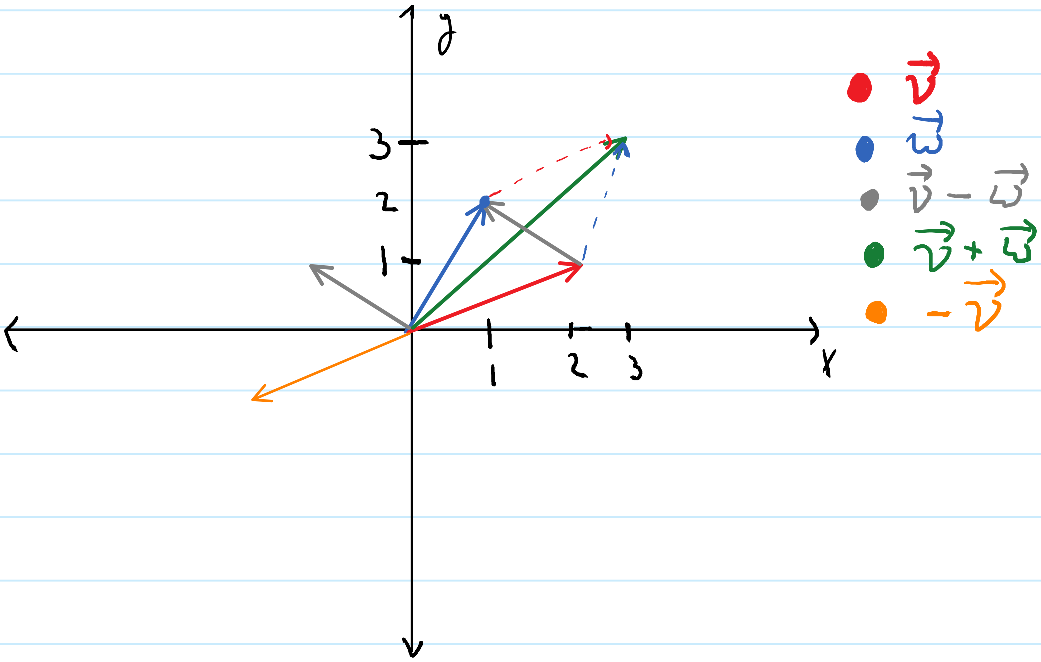
$\boxed{5}$

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$\boxed{8}$ Similar to 5, but in $\mathbb{R}^3$.

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$\boxed{16}$ To describe the line, we just need:

1. A point, $P_0$, on the line, and
2. A "direction vector", $\vec{d}$, for the line.

Here, $P_0 = (0, 2, 1)$, $\vec{d} = 2\vec{i} - \vec{k}$
$$= (2, 0, -1).$$

Therefore, the line can be described by:

(vector form): $\vec{\ell}(t) = \vec{OP_0} + t\vec{d}$
$$\vec{\ell}(t) = (0, 2, 1) + t(2, 0, -1)$$

(parametric eqn form):

$$x = 2t$$
$$y = 2$$
$$z = 1 - t$$

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$\boxed{18}$ This time $P_0 = (-5, 0, 4)$ (or $P_0 = (6, -3, 2)$).

For the direction vector, we can use the vector from $(-5, 0, 4)$ to $(6, -3, 2)$:

$$\vec{d} = (6, -3, 2) - (-5, 0, 4)$$
$$= (11, -3, -2)$$

Then the line is given in vector form by

$$\vec{\ell}(t) = \vec{OP_0} + t\vec{d}$$
$$\boxed{\vec{\ell}(t) = (-5, 0, 4) + t(11, -3, -2).}$$

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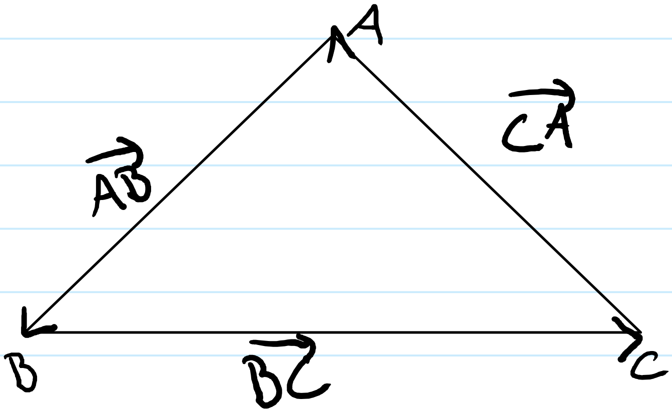
$\boxed{23}$ Let's draw the picture in $\mathbb{R}^2$ since it is easier to visualize.




Use the "head to tail" geometric description of vector addition: We see that the vector $\vec{AB} + \vec{BC} + \vec{CA}$ starts and ends at the point $A$. Since the head and tail of this vector are the same, we have $\vec{AB} + \vec{BC} + \vec{CA} = \vec{0}$.

$\boxed{24}$ $\vec{\ell}(t)$ intersects the xy-plane when the z-component is 0. This happens when

$$-2+t=0 \Rightarrow t=2.$$

So The line intersects the xy-plane at

$$\vec{\ell}(2) = (3+2(2),\ 7+8(2),\ -2+2)$$
$$= \boxed{(7, 23, 0)}$$

To find the intersection with the xz-plane (resp. yz-plane), set $y=0$ (resp. $x=0$) and do a similar thing as above.

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$\boxed{26}$ Every point on the line satisfies:

$$\left.\begin{aligned} x &= 1+2t \\ y &= -1+3t \\ z &= 2+t \end{aligned}\right\}$$

(switch to parametric eqns.)

Substitute these into the expression:

$5x-3y-z-6=0$, and see what happens.

$$5(1+2t)-3(-1+3t)-(2+t)-6=0$$
$$\Leftrightarrow \quad 5+10t+3-9t-2+t-6=0$$
$$\Leftrightarrow \quad (5+3-2-6)+(10-9+1)t=0$$
$$\Leftrightarrow \quad 0=0$$

Since this is true for all real #s, $t$, we are done.

$\boxed{28}$ If the lines intersect, then we can find a t-value and an s-value s.t.:

$$(\underbrace{t+4}_{x}, \underbrace{4t+5}_{y}, \underbrace{t-2}_{z}) = (\underbrace{2s+3}_{x}, \underbrace{s+1}_{y}, \underbrace{2s-3}_{z}).$$

This implies:
① $t+4 = 2s+3$
② $4t+5 = s+1$
③ $t-2 = 2s-3$

Using ③: $t-2 = 2s-3 \Rightarrow \boxed{t = 2s-1}$

plug into ②:
$$4t+5 = s+1 \Rightarrow$$
$$4(2s-1)+5 = s+1 \Rightarrow$$
$$8s-4+5 = s+1 \Rightarrow$$
$$7s+1 = 1 \Rightarrow$$
$$\boxed{s=0}$$

plug back into $t = 2s-1$, we get $\boxed{t=-1}$

This shows that the y-coords and the z-coords match when $\boxed{s=0, t=-1}$.

We plug into ① to see if the x-coords also match when $s=0$, $t=-1$.

$$t+4 = 2s+3 \Rightarrow$$
$$-1+4 = 2(0)+3 \Rightarrow$$
$$3 = 3 \quad \checkmark$$

Therefore, the lines intersect when $s=0$, $t=-1$.
The point of intersection is then:

$\boxed{t=-1}$:
$$(t+4, 4t+5, t-2) = (-1+4, 4(-1)+5, -1-2)$$
$$= \boxed{(3, 1, -3)}$$

§1.2: 4, 9, 11, 13, 15, 17, 20, 23, 29

Hint: the identitys $\cancel{\vec{u}\cdot\vec{v} = \|\vec{u}\|\|\vec{v}\|\cos\theta}$ $\boxed{\vec{u}\cdot\vec{u} = \|\vec{u}\|^2}$ are crucial!

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$\boxed{4}$ If $\vec{v} = \dfrac{\vec{u}}{\|\vec{u}\|}$, we have:

$$\vec{u}\cdot\vec{v} = \vec{u}\cdot\left(\frac{\vec{u}}{\|\vec{u}\|}\right) = \frac{1}{\|\vec{u}\|}(\vec{u}\cdot\vec{u})$$

$$= \frac{\|\vec{u}\|^2}{\|\vec{u}\|}$$

$$= \|\vec{u}\|$$

Now since $\vec{u} = \sqrt{3}\,\vec{i} - 315\vec{j} + 22\vec{k}$
$= (\sqrt{3}, -315, 22)$,

we have: $\vec{u}\cdot\vec{v} = \|\vec{u}\| = \left((\sqrt{3})^2 + (-315)^2 + (22)^2\right)^{1/2}$
$\boxed{= \sqrt{99712}}$

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$\boxed{9}$ $\vec{u} = (-1, 3, 1)$, $\vec{v} = (-2, -3, -7)$.

- $\|\vec{u}\| = \sqrt{(-1)^2 + 3^2 + 1^2} = \boxed{\sqrt{11}}$
- $\|\vec{v}\| = \sqrt{(-2)^2 + (-3)^2 + (-7)^2} = \boxed{\sqrt{62}}$
- $\vec{u}\cdot\vec{v} = (-1)(-2) + (3)(-3) + (1)(-7)$
$= 2 - 9 - 7$
$\boxed{= -14}$

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$\boxed{11}$ $\|\vec{u}\| = \boxed{\sqrt{14}}$
$\|\vec{v}\| = \boxed{\sqrt{26}}$
$\vec{u}\cdot\vec{v} = \boxed{-17}$

**13** $\boxed{\text{Recall: } \vec{u} \perp \vec{v} \text{ iff } \vec{u}\cdot\vec{v} = 0}$

We want to find $b, c$ values for which

① $(5, b, c)\cdot(1,2,3) = 0$ and
② $(5, b, c)\cdot(1,-2,1) = 0$

From ①, we get: $5 + 2b + 3c = 0 \Rightarrow$

$$\boxed{2b + 3c = -5}$$

From ②: $5 - 2b + c = 0 \Rightarrow$

$$\boxed{-2b + c = -5}$$

Solve this system of eqns:

Add the equations to get $4c = -10$

$$\Rightarrow \boxed{c = \frac{-5}{2}}$$

plug back into either eqn to get

$$\boxed{b = \frac{5}{4}}$$

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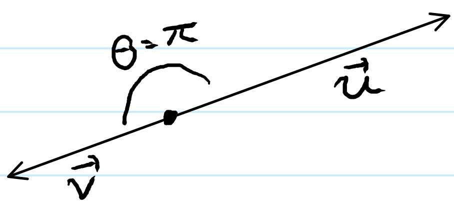
**15** Since $\vec{u}\cdot\vec{v} = \|\vec{u}\|\,\|\vec{v}\|\cos(\theta)$ where $\theta$ is the angle between $\vec{u}$ and $\vec{v}$,
and since $\vec{u}\cdot\vec{v} \overset{!}{=} -\|\vec{u}\|\,\|\vec{v}\|$,
we have:

$$\cos\theta = -1 \Rightarrow \theta = \pi$$




$\vec{u}$ and $\vec{v}$ point in opposite directions

**17** Let's just find the angle between the vectors in #9.

We have (from before) $\boxed{\begin{aligned}\|\vec{u}\| &= \sqrt{11}\\ \|\vec{v}\| &= \sqrt{62}\\ \vec{u}\cdot\vec{v} &= -14\end{aligned}}$

Using $\vec{u}\cdot\vec{v} = \|\vec{u}\|\,\|\vec{v}\|\cos\theta$, we see,

$$-14 = (\sqrt{11})(\sqrt{62})\cos\theta \Rightarrow$$

$$\boxed{\theta = \cos^{-1}\left(\frac{-14}{(\sqrt{11})(\sqrt{62})}\right)}.$$

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**20** Recall: $\text{proj}_{\vec{v}}(\vec{u}) = \left(\frac{\vec{u}\cdot\vec{v}}{\vec{v}\cdot\vec{v}}\right)\vec{v}.$

We have: $\vec{u}\cdot\vec{v} = -4$

$\vec{v}\cdot\vec{v} = 14.$

$$\Rightarrow \text{proj}_{\vec{v}}(\vec{u}) = \left(\frac{-4}{14}\right)(2,1,-3)$$

$$\boxed{= \left(\frac{-4}{7}, \frac{-2}{7}, \frac{6}{7}\right)}$$

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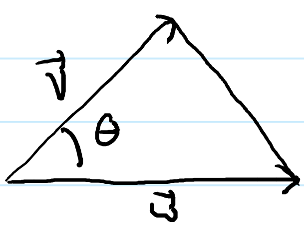
**23**




If the $\Delta$ is equilateral, $\theta = 60^\circ$. Since $\|\vec{v}\| = \|\vec{w}\| = 1$, we have

$$\vec{v}\cdot\vec{w} = \|\vec{v}\|\,\|\vec{w}\|\cos\theta$$

$$= (1)(1)\cos(60^\circ)$$

$$\boxed{= \tfrac{1}{2}}$$

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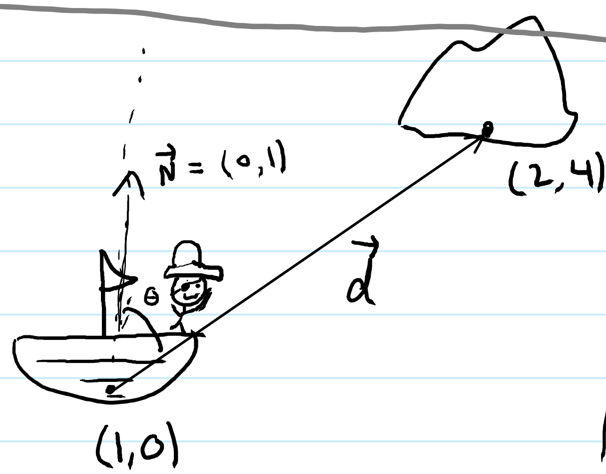
**29**




Here, $\vec{N}$ is a vector pointing north. So let $\vec{N} = (0,1)$. $\vec{d}$ is the vector from $(1,0)$ to $(2,4)$.

so $\vec{d} = (2,4) - (1,0)$

$= \boxed{(1,4)}.$

To find $\theta$, use

$$\vec{N} \cdot \vec{d} = \|\vec{N}\| \, \|\vec{d}\| \cos\theta \Rightarrow$$

$$4 = (1)(\sqrt{17}) \cos\theta \Rightarrow$$

$$\boxed{\theta = \cos^{-1}\left(\frac{4}{\sqrt{17}}\right)}$$