Math 3c Midterm 1 topics and sample questions

October 15, 2016

1 Reminders:

Reading the textbook is considered part of your homework and is considered mandatory (see the syllabus). You are responsible for reading the material in the book whether or not it was discussed in lecture[1]. However, the material we talked about in lecture is what I believe to be the most important topics in this course. I mention this because there are great example questions in the textbook that I do not have the time to do in class.

Secondly, exams are designed to test whether or not you can solve problems using what you've learned. With this in mind, you may see types problems on my exams that you may not have seen before. Nevertheless, if you understand the material, you should be able to apply your knowledge to come up with the correct solution. Lastly, the difficulty level of these practice problems does not necessarily represent the difficulty of the exam. Some of these questions are easy, some of these questions are difficult.

Chapter 1

1.3

- Inequalities
- Sets/Intevals
- Absolute Value

Problems:

1. Solve for x: $|3x - 9| = 11$.
2. Write the following set as a single interval
$$\{x : x \geq -1\} \cup \{x : -2 \leq x < 5\}.$$
3. Find all values of x that satisfy the inequality
$$|7x - 1| > 4.$$
Express your answer as an interval, or as a union of intervals.

[1] Skip the discussion in §2.3 about ellipses and hyperbolas.

4. Write the definition of absolute value.

5. Express the following set as an interval, or as a union of intervals:

$$\{x : |x - 2| < 3\}.$$

6. Find all values of x such that

$$|2x^2 - 14x + 30| \leq 0.$$

Express your answer as an interval, or as a union of intervals.

7. Find all values of x such that

$$\left| \frac{x - 1}{2x + 1} \right| = 5.$$

Hint: there are two such values of x.

8. Explain why the function $f(x) = | - 8x + 2|$ is not one to one. Find an explicit example of when $f(a) = f(b)$, but $a \neq b$ (which would prove, by definition, that f is not one to one.)

Chapter 2

2.1

- Distance Formula.
- The coordinate Plane.

2.2

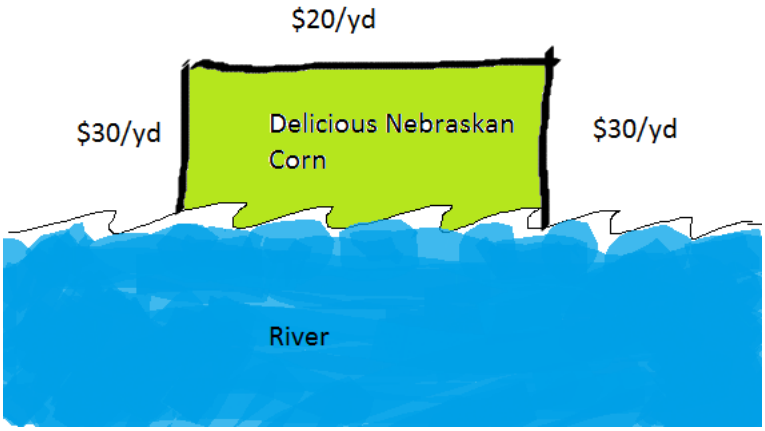
- Slope.
- Lines: Point-slope form and slope-intercept.
- Parallel and Perpendicular lines.
- Midpoints.

2.3

- Completing the square.
- The quadratic formula.
- Parabolas.
- Vertex form of a parabola.
- Equation of circles.

Problems:

1. Find the distance between the points $(-2, 3)$ and $(-11, 2)$.
2. Find the midpoint between the points $(-2, 3)$ and $(-11, 2)$.
3. Find the slope of the line that passes through the points $(-2, 3)$ and $(-11, 2)$.
4. Put the parabola $y = x^2 - 14x + 21$ in vertex form. I.e., $y = d(x - h)^2 + k$.
5. Find the vertex of the parabola $y = -3x^2 - 11x + 7$.
6. Find the roots of the parabola $y = -3x^2 - 11x + 7$.
7. Sketch a graph of the parabola $y = -3x^2 - 11x + 7$.
8. Write an equation that describes the set of all points in the $xy - plane$ that is a distance of 9 from the point $(13, -17)$.
9. Find an equation for the line that passes through the points $(-2, 3)$ and $(-11, 2)$. Express your answer in both point-slope form and slope-intercept form.
10. Write the equation of a line that is parallel to the line $y = \frac{1}{9}x + \frac{29}{9}$.
11. Write the equation of a line that is perpendicular to the line $y = \frac{1}{9}x + \frac{29}{9}$.
12. Put the parabola $y = -3x^2 - 24x - 5$ in vertex form.
13. Find the center and radius of the circle defined by the equation
$$2x^2 - 8x + 2y^2 + 12y = 24.$$
14. Find the point on the line $y = 2x - 2$ that is closest to the origin.
15. A farmer from Omaha, Nebraska throws a ball straight into the air. Its height in feet is given by the function $h(t) = -16.1t^2 + 60t + 5$ where t represents time in seconds. What is the maximal height achieved by the ball? When does the ball reach its maximum height. How long does it take until the ball hits the ground? What color is the ball?
16. The sum of two numbers a and b is 5. The product of a and b is 10. What are a and b? Your answer will not be an integer.
17. A farmer from Omaha Nebraska is building a fence to protect his rectangular corn field. One side of the corn field lies against a river bed, and therefore the farmer does not need any fencing on this side of the field. The fencing that lies perpendicular to the river bed costs \$30 per yard to construct, while the fencing that is parallel to the river bed costs \$20 per yard to construct. If the farmer has a budget of \$500. What is the largest area he can enclose with the fence?



Chapter 3

3.1

- Functions.
- Domain.
- Range.
- Graphs

3.2

- Graph Transformations.
- Even Functions.
- Odd Functions.

3.3

- Creating new functions out of old. I.e., Adding/subtracting/multiplying/dividing functions.
- Function Composition.
- Noncommutativity of function composition. I.e., $f \circ g \neq g \circ f$.

3.4

- Inverse Functions.
- One to one functions.
- Domain and Range of inverse functions.
- The identity function.

3.5

- Graph of a function vs. its inverse.

Problems:

1. Does $y = x^2 - 1$ represent a function? How do you know?
2. Does $y = \pm\sqrt{x+1}$ represent a function? How do you know?
3. Define the identity function.
4. Let $f(x) = x^3 + 1$, $g(x) = \sqrt{x^2 - 2x + 1}$, $h(x) = \frac{x-1}{x+1}$. Write an equation for each of the following functions and find its domain.
   (a) g.
   (b) h.
   (c) $f \circ g$.
   (d) $g \circ f$.
   (e) $h \circ g$.
   (f) $h \circ h$.
   (g) $f + g$.
   (h) fg.
   (i) g/f.
   (j) f^{-1} if it exists. If it does not exist, explain why.
   (k) h^{-1} if it exists. If it does not exist, explain why.
   (l) g^{-1} if it exists. If it does not exist, explain why.
5. Let $y = x^2$. Find formulae for:
   (a) the graph of $y = x^2$ shifted up by 5,
   (b) the graph of $y = x^2$ shifted right by 5,
   (c) the graph of $y = x^2$ stretched vertically by 5,
   (d) the graph of $y = x^2$ stretched horizontally by 5.
6. Sketch:
   (a) $y = -2x^2 - 1$.
   (b) $y = -2(x^2 - 1)$.
   (c) $y = (x + 2)^2 - 1$.

Chapter 4

4.1

- Exponent Rules

4.2

- Polynomial functions.
- Degree.
- Roots of polynomials.
- "End behavior" Polynomials. I.e., what happens to $p(x)$ when $|x| \gg 0$.

4.3

- Rational functions.
- Polynomial division.
- "End behavior" of rational functions (horizontal asymptotes).
- Domain of rational functions (vertical asymptotes).

Problems:

1. Write the following expression as a single power of 2:
$$\left(\frac{(\sqrt{2})^8(16^2)(4^{-2})}{(\sqrt{32})^3 2^{-2}}\right)^{-2}.$$
2. Find the quotient and remainder of $14x^5 + 3x - 11$ after division by $2x^2 - 1$.
3. What is the degree of the polynomial $p(x) = 3(x-1)(x^2-4)(x^2+5)$.
4. Find the roots of the polynomial $p(x) = 3(x-1)(x^2-4)(x^2+5)$.
5. What happens to $p(x) = 3(x-1)(x^2-4)(x^2+5)$ as $x \to \pm\infty$. In other words, what does $p(x)$ do as x gets bigger and bigger (or smaller and smaller negative).
6. Sketch a graph of $q(x) = 3(x-1)(x^2-4)(x^2-5)$.
7. Find the zeroes of the rational function $f(x) = \frac{3x^2-x+3}{x^2-9}$.
8. What is the domain of $f(x) = \frac{3x^2-x+3}{x^2-9}$?
9. Find the vertical and horizontal asymptotes of $f(x) = \frac{3x^2-x+3}{x^2-9}$.