a.

1. P_0 = initial value
= 30 (mg)

Since the half-life is $\boxed{6 \text{ years}}$, after 6-years you have half as much as you started with

Therefore, we have

$$P(6) = \frac{1}{2}(30)$$

So
$$30\, b^6 = \frac{1}{2}(30)$$
$$\Rightarrow \quad b^6 = \frac{1}{2}$$
$$\Rightarrow \quad \boxed{b = \left(\frac{1}{2}\right)^{\frac{1}{6}}}$$

So
$$P(t) = 30\left(\frac{1}{2}\right)^{\frac{t}{6}}$$

1. b. We need to solve

$$P(t) = \underline{(0.05)}\,\underline{(30)}$$

5% of initial value

So
$$\cancel{30}\left(\frac{1}{2}\right)^{\frac{t}{6}} = (0.05)(\cancel{30})$$
$$\left(\frac{1}{2}\right)^{\frac{t}{6}} = 0.05$$
$$\frac{t}{6} = \log_{\frac{1}{2}}(0.05)$$
$$t = 6\log_{\frac{1}{2}}(0.05)$$

2a. $2\log_x(2) + \log_x(9) = 2$

$\Rightarrow \log_x(2^2) + \log_x(9) = 2$

$\Rightarrow \log_x(4) + \log_x(9) = 2$

$\Rightarrow \log_x(36) = 2$

$\Rightarrow x^2 = 36$

$\Rightarrow \boxed{x = 6}$

(Notice x can't be negative because the base of a log is >0 in this class).

2b) $e^{2x} + e^x = 6$

$e^{2x} + e^x - 6 = 0$

$(e^x)^2 + e^x - 6 = 0$ Let $u = e^x$

$u^2 + u - 6 = 0$

$(u+3)(u-2) = 0$

$u = -3$ or $u = 2$ (substitute back)

$e^x = -3$ or $e^x = 2$

This is impossible

$\Rightarrow \boxed{x = \ln(2)}$

3a) $-x + 4y = -4$

$2x + y = 0$

Using substitution, $-x + 4y = -4$

$\Rightarrow 4y = x - 4$

$\Rightarrow \boxed{x = 4y + 4}$

$2x + y = 0$

$\Rightarrow \quad 2(\underbrace{4y+4}_{\text{Substitute!}}) + y = 0$

$\Rightarrow \quad 8y + 8 + y = 0$

$\Rightarrow \quad 9y + 8 = 0$

$\Rightarrow \quad 9y = -8$

$y = \frac{-8}{9}$

$\Rightarrow \quad x = 4\left(\frac{-8}{9}\right) + 4\left(\frac{9}{9}\right)$

$= \frac{-32}{9} + \frac{36}{9}$

$= \frac{4}{9}$

One sol. $\boxed{\left(\frac{4}{9}, \frac{-8}{9}\right)}$

(saying $x = \frac{4}{9}$ and $y = \frac{-8}{9}$ is fine).

3b) No, a system of two linear eq'ns with two variables can have either

0 solutions
1 solution or
infinitely many solutions

(4a) To convert from deg. to rad. multiply by

$$\frac{\pi}{180}.$$

$$540° = 540°\left(\frac{\pi \text{ rad}}{180°}\right) = 3\pi \text{ rad.}$$

(both numerator and denominator are divisible by 90)

(4b) To go from rad. to deg., multiply by

$$\frac{180°}{\pi \text{ rad.}}$$

$$\left(\frac{11\pi}{12}\text{ rad.}\right)\left(\frac{180°}{\pi \text{ rad.}}\right) = \left(\frac{11}{\cancel{12}}\right)\left(\overset{15}{\cancel{180}}\right) = (11)(15)$$

$$= 165°$$

(5a) Pythagoras!

$$\sin^2\theta + \cos^2\theta = 1$$

Since $\sin(\theta) = \frac{2}{3}$,

$$\left(\frac{2}{3}\right)^2 + \cos^2(\theta) = 1$$

$$\frac{4}{9} + \cos^2(\theta) = 1$$

$$\cos^2(\theta) = 1 - \frac{4}{9}$$

$$\cos^2(\theta) = \frac{5}{9}$$

$$\cos(\theta) = -\sqrt{\frac{5}{9}}$$

(Since $\cos\theta < 0$, by assumption, we know $\cos(\theta) = -\sqrt{\frac{5}{9}}$, not $\sqrt{\frac{5}{9}}$)

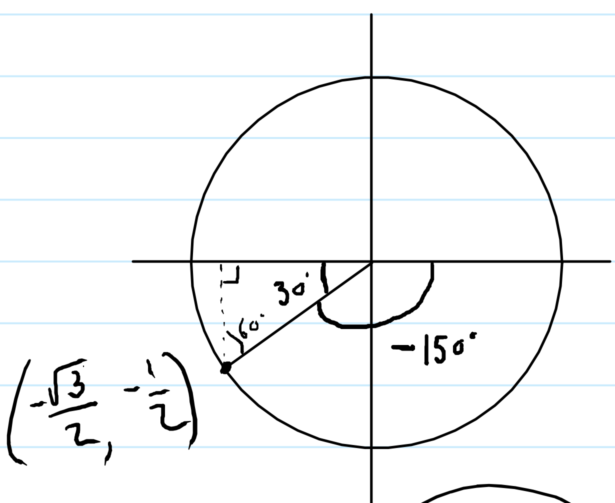
$$\cos(\theta) = -\frac{\sqrt{5}}{3}$$

(5b) Draw the radius corresponding to $\frac{-5\pi}{6}$ radians

Y6 Warning: If it doesn't specify a unit, assume its in radians

Also, $\frac{-5\pi}{6} = -150°$

The "reference" angle is 30°, so the triangle in the picture is a "30°, 60°, 90°" triangle.



The point corresponding to $-\frac{5\pi}{6}$ is $\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

So $\cos(\theta) = -\frac{\sqrt{3}}{2}$, $\sin(\theta) = -\frac{1}{2}$, $\tan(\theta) = \frac{-\frac{1}{2}}{-\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}}$